Equidispersion and moment conditions for count panel data model*

Yoshitsugu Kitazawa†

June 5, 2009

Abstract
This paper proposes some new moment conditions under the assumption of the equidispersion in count panel data model. These are obtained by using the association between variances and covariances in the disturbance. Some Monte Carlo experiments configured for the Poisson model show that the GMM estimators using the new moment conditions perform better than the conventional quasi-differenced GMM estimator and some gains are recognized in using the new moment conditions.

Keywords: count panel data, linear feedback model, equidispersion, implicit operation, cross-linkage moment conditions, GMM, Monte Carlo experiments

JEL classification: C23

1. Introduction

Assuming the equidispersion is a traditional way in count panel data model. It implies the equality of mean and variance of the dependent variable, which is characteristic of the Poisson regression. Hausman et al. (1984) use the Poisson conditional maximum likelihood estimator (CMLE) with the intention of analyzing count panel data, taking the fixed effect into consideration. However, this estimator is consistent only for the model with strictly exogenous explanatory variables and no dynamics, when number of individuals is large and number of time periods is small. In addition, the CMLE is identical to the ordinary maximum likelihood estimator for the Poisson model and further results in the within group (WG) mean scaling estimator, which requires no distributional assumption, not to mention the assumption of the equidispersion (see Blundell et al., 2002 and Lancaster, 2002). After all, the traditional and famous estimators for the case of strictly exogenous explanatory variables in count panel data model necessitate no assumption of the equidispersion.

In count panel data model, it is much acceptable to regard the explanatory variables as being predetermined instead of being strictly exogenous. In the case of assuming the predetermined explanatory variables, the generalized method of moments (GMM) estimators proposed by Hansen (1982) are exclusively utilized by using the moment conditions proposed by Chamberlain (1992), Wooldridge (1997), Windmeijer (2000), Blundell et al. (2002) and Kitazawa (2007), except for the case where the pre-sample mean (PSM) estimator proposed by Blundell et al. (1999, 2002) is usable. However, no moment condition is proposed associated with the equidispersion, except for Kitazawa (2007).

As stated above, the discussion on the equidispersion has been substantively neglected in the

* Discussion Paper Series, Faculty of Economics, Kyushu Sangyo University Discussion Paper, June 2009, No. 33. Errata information: http://www.ip.kyusan-u.ac.jp/J/kitazawa/ERRATA/errata_equi.html

† Correspondence: Faculty of Economics, Kyushu Sangyo University, Matsukadai 2-3-1, Higashi-ku, Fukuoka, Japan. E-mail: kitazawa@ip.kyusan-u.ac.jp

mainstream of count panel data model.[1] However, it is conceivable that the examination of probability distributions for count dependent variables is meaningful for the development of count panel data econometrics. Whether or not the equidispersion is acceptable has not been fully corroborated in count panel data model incorporating individual effects (and time dummies). To contribute to the advancement of the discussion on the equidispersion, this paper proposes the newly reformed moment conditions associated with the equidispersion in the framework of the linear feedback model (LFM) proposed by Blundell et al. (2002).

Although some moment conditions associated with the equidispersion are proposed by Kitazawa (2007), this paper proposes some new moment conditions associated with the equidispersion, which are obtained by setting up relationships between variances and covariances in the disturbances in the LFM. The new moment conditions are referred to as the "cross-linkage moment conditions" in this paper, named after the construction of the bridge between variances and covariances in the disturbances. Then, the implicit operation proposed by Kitazawa (2007) is used for the construction of the cross-linkage moment conditions.[2] The advantage of the new moment conditions is that they contain a large number of moment conditions linear with respect to the coefficient on lagged count dependent variables in the LFM.

The Monte Carlo experiments are carried out in the same configuration as in Blundell et al. (2002), except for the setting of the initial condition of dependent variables. The experiments show that the GMM estimators using the newly proposed moment conditions associated with the equidispersion perform well, especially compared to the conventional quasi-differenced GMM estimator.

The paper is organized as follows. In section 2, the cross-linkage moment conditions are proposed. In section 3, some Monte Carlo experiments investigate the small sample properties of the GMM estimators using the cross-linkage moment conditions. Section 4 concludes.

2. Model, moment conditions and GMM estimators

In this section, the new sets of the moment conditions associated with the equidispersion for the linear feedback model (LFM) proposed by Blundell et al. (2002) in count panel data are proposed for the three cases: the case of predetermined explanatory variables, the case of strictly exogenous explanatory variables and the case of mean-stationary dependent variables. The method of deriving these sets is based on the implicit operation proposed by Kitazawa (2007). The GMM estimators are constructed by using the cross-linkage moment conditions.

2.1. Linear feedback model

A simple form of the linear feedback model (LFM) proposed by Blundell et al. (2002) is as follows:

$$y_{it}=\gamma y_{i,t-1}+\exp(\beta x_{it}+\eta_i)+v_{it} \ , \qquad \text{for} \quad t=2,\dots,T \ , \tag{2.1.1}$$

where the subscript i denotes the individual unit with $i=1,\dots,N$, t denotes the time period and it is assumed that T is fixed and $N\rightarrow\infty$. The count dependent variable y_{it} is able to have zero or positive integer values and the explanatory variable x_{it} is able to have the real number. The unobservable variables η_i and v_{it} are the individual specific effect and the disturbance respectively. The parameters of interest are γ (with $|\gamma|<1$) and β .

Equation (2.1.1) is rewritten as follows:

1 The discussion on the overdispersion is conducted since Hausman et al. (1984).

2 In the context of the ordinary dynamic panel data model, Ahn (1990) and Ahn and Schmidt (1995) propose the method of constructing the efficient sets of the moment conditions. The implicit operation is developed for the purpose of incorporating their method into count panel data model.

$$y_{it}=\gamma y_{i,t-1}+u_{it} \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \qquad (2.1.2)$$

$$u_{it}=\phi_i \mu_{it}+v_{it} \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \qquad (2.1.3)$$

where $\phi_i=\exp(\eta_i)$ and $\mu_{it}=\exp(\beta x_{it})$. Based on (2.1.2), it can be seen that u_{it} is observable in the sense that it is written in terms of data and parameter. That is, $u_{it}=y_{it}-\gamma y_{i,t-1}$, which is plugged into the moment conditions to be hereinafter described.

2.2. Case of predetermined explanatory variables

In this case, the assumption on the disturbance v_{it} is

$$E[v_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \qquad (2.2.1)$$

where $v_i^{t-1}=(v_{i1},\ldots,v_{i,t-1})$ and $x_i^t=(x_{i1},\ldots,x_{it})$. The assumption (2.2.1) is referred to as the "original assumption" for the case of predetermined explanatory variables. Kitazawa (2007) constructs the implicit standard assumptions from the original assumption (2.2.1) as follows:

$$E[y_{i1}v_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad , \qquad (2.2.2)$$

$$E[v_{is}v_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad , \quad \text{for} \quad 2\le s\le t-1 \quad , \qquad (2.2.3)$$

$$E[x_{is}v_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad , \quad \text{for} \quad 1\le s\le t \quad , \qquad (2.2.4)$$

$$E[\eta_i v_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad . \qquad (2.2.5)$$

Here, the following assumption with respect to the equidispersion is imposed in addition to the implicit standard assumptions (2.2.2) – (2.2.5):

$$E[(v_{it}^2-y_{it}) \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad . \qquad (2.2.6)$$

Using the assumptions (2.2.2) – (2.2.6) (and quoting the original assumption (2.2.1) sporadically), new types of moment conditions based on the relationships holding between variances and covariances in the disturbances are proposed in addition to the conventional quasi-differenced moment conditions proposed by Chamberlain (1992) and Wooldridge (1997) for the case of predetermined explanatory variables in this section. The line of constructing the moment conditions in manner of Kitazawa (2007) is sketched as below.

According to Kitazawa (2007), the observable analogues for (2.2.2), (2.2.3), (2.2.4), and (2.2.6) are obtained by replacing the unobservable variables v_{it} by the observable variables u_{it} :

$$E[y_{i1}u_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=y_{i1}\phi_i\mu_{it} \quad , \qquad (2.2.7)$$

$$E[u_{is}u_{it} \mid y_{i1},\eta_i,v_i^{t-1},x_i^t]=\phi_i^2\mu_{is}\mu_{it}+v_{is}\phi_i\mu_{it} \quad , \qquad \text{for} \quad 2\le s\le t-1 \quad , \qquad (2.2.8)$$

$$E[x_{is}u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = x_{is}\phi_i\mu_{it} \quad , \qquad \text{for} \quad 1 \leq s \leq t \quad , \tag{2.2.9}$$

$$E[(u_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^t] = \phi_i^2 \mu_{it}^2 \quad , \tag{2.2.10}$$

respectively.

Using the observable analogues (2.2.7) – (2.2.10), the relationships holding among $y_{i1}u_{it}$ for $t=2,\ldots,T$, among $u_{is}u_{it}$ for $s=2,\ldots,t$ and $t=2,\ldots,T$ and among $x_{is}u_{it}$ for $s=1,\ldots,t$ and $t=2,\ldots,T$ are solved through the intermediary of the unconditional expectation operator after weighting them with appropriate transformations of explanatory variables x_{it} for $t=1,\ldots,T$. With the intention of ruling out the redundancies of the moment conditions to be constructed, condensed full sets of the relationships above are used to construct a lean set of the moment conditions.

A condensed full set of the relationships among $y_{i1}u_{it}$ for $t=2,\ldots,T$ is composed of the relationships between $y_{i1}u_{i,t-1}$ and $y_{i1}u_{it}$ for $t=3,\ldots,T$. The other relationships among $y_{i1}u_{it}$ for $t=2,\ldots,T$ are indirectly traced based on the trunk connections by exploiting the condensed full set.

In addition, a condensed full set of the relationships among $u_{is}u_{it}$ for $s=2,\ldots,t$ and $t=2,\ldots,T$ is composed of the relationships between $u_{is}u_{i,t-1}$ and $u_{is}u_{it}$ for $s=2,\ldots,t-1$ and $t=3,\ldots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\ldots,T$. The other relationships among $u_{is}u_{it}$ for $s=2,\ldots,t$ and $t=2,\ldots,T$ are indirectly traced based on the trunk connections by exploiting the condensed full set.

Both relationships between $y_{i1}u_{i,t-1}$ and $y_{i1}u_{it}$ for $t=3,\ldots,T$ and between $u_{is}u_{i,t-1}$ and $u_{is}u_{it}$ for $s=2,\ldots,t-2$ and $t=4,\ldots,T$ are solved by Kitazawa (2007) in order to construct the $(T-2)(T-1)/2$ moment conditions based on the product of the (observable) instrument variables and the quasi-differenced transformation proposed by Chamberlain (1992) and Wooldridge (1997), which are regarded as the application to the LFM by Blundell et al. (2002). That is,

$$E[y_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0 \quad , \qquad \text{for} \quad s=1,\ldots,t-2 \quad ; \quad t=3,\ldots,T \quad , \tag{2.2.11}$$

which hold even if the assumption (2.2.6) is not imposed.

Accordingly, in this paper, the relationships between $u_{i,t-1}^2$ and $u_{i,t-1}u_{it}$ for $t=3,\ldots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\ldots,T$ are solved in order to construct the moment conditions valid for the case of predetermined explanatory variables. These moment conditions are referred to as the cross-linkage moment conditions in this paper, because they represent the relationships bridging between the observable analogues of variances of v_{it} and the observable analogues of covariances of v_{it} in the case of the equidispersion.

Further, the $(T-1)T/2-1$ moment conditions based on a condensed full set of the relationships among $x_{is}u_{it}$ for $s=1,\ldots,t$ and $t=2,\ldots,T$ are found by Kitazawa (2007). They are

$$E[x_{is}((\mu_{i,t-1}/\mu_{it})u_{it} - u_{i,t-1})] = 0 \quad , \qquad \text{for} \quad s=1,\ldots,t-1 \quad ; \quad t=3,\ldots,T \quad , \tag{2.2.12}$$

which are obtained by solving the relationships between $x_{is}u_{i,t-1}$ and $x_{is}u_{it}$ for $s=1,\ldots,t-1$ and $t=3,\ldots,T$. These moment conditions are also those based on the product of the (observable) instrument variables and the quasi-differenced transformation proposed by Chamberlain (1992) and Wooldridge (1997).

From now on, two types of the cross-linkage moment conditions are solved by using the relationships between $u_{i,t-1}^2$ and $u_{i,t-1}u_{it}$ for $t=3,\ldots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\ldots,T$.

First, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{i,t-1}^2$ and $u_{i,t-1}u_{it}$ (weighted with $\mu_{i,t-1}/\mu_{it}$). Multiplying both sides of (2.2.8) for $s=t-1$ by $\mu_{i,t-1}/\mu_{it}$ gives

$$E[u_{i,t-1}(\mu_{i,t-1}/\mu_{it})u_{it}\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=\phi_i^2\mu_{i,t-1}^2+v_{i,t-1}\phi_i\mu_{i,t-1} \quad . \tag{2.2.13}$$

Applying the law of total expectation to (2.2.10) dated $t-1$ and (2.2.13), it follows that

$$E[(u_{i,t-1}^2-y_{i,t-1})]=E[\phi_i^2\mu_{i,t-1}^2] \quad , \tag{2.2.14}$$

$$E[u_{i,t-1}(\mu_{i,t-1}/\mu_{it})u_{it}]=E[\phi_i^2\mu_{i,t-1}^2] \quad . \tag{2.2.15}$$

Subtracting (2.2.14) from (2.2.15) gives

$$E[u_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})+y_{i,t-1}]=0 \quad . \tag{2.2.16}$$

At this stage, creating the recursive equations

$$\begin{aligned}&E[y_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})]\\&=\gamma E[y_{i,t-2}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})]+E[u_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})]\end{aligned} \tag{2.2.17}$$

from (2.1.2) dated $t-1$ and applying the moment conditions (2.2.11) for $s=t-2$, it can be seen that the following relationships hold:

$$E[u_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})]=E[y_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})] \quad . \tag{2.2.18}$$

Accordingly, plugging (2.2.18) into (2.2.16) gives the following $T-2$ cross-linkage moment conditions:

$$E[y_{i,t-1}((\mu_{i,t-1}/\mu_{it})u_{it}-(u_{i,t-1}-1))]=0 \quad , \quad \text{for} \quad t=3,\ldots,T \quad , \tag{2.2.19}$$

in which the order reduction with respect to γ is realized, compared to (2.2.16).

Next, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{i,t-1}u_{it}$ (weighted with $1/\mu_{it}$) and u_{it}^2 (weighted with $\mu_{i,t-1}/\mu_{it}^2$). Multiplying (2.2.8) for $s=t-1$ by $1/\mu_{it}$ gives

$$E[u_{i,t-1}u_{it}(1/\mu_{it})\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=\phi_i^2\mu_{i,t-1}+v_{i,t-1}\phi_i \tag{2.2.20}$$

and multiplying (2.2.10) by $\mu_{i,t-1}/\mu_{it}^2$ gives

$$E[(u_{it}^2-y_{it})(\mu_{i,t-1}/\mu_{it}^2)\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=\phi_i^2\mu_{i,t-1} \quad . \tag{2.2.21}$$

Applying the law of total expectation to (2.2.20) and (2.2.21), it follows that

$$E[u_{i,t-1}u_{it}(1/\mu_{it})]=E[\phi_i^2\mu_{i,t-1}] \quad , \tag{2.2.22}$$

$$E[(u_{it}^2-y_{it})(\mu_{i,t-1}/\mu_{it}^2)]=E[\phi_i^2\mu_{i,t-1}] \quad . \tag{2.2.23}$$

Subtracting (2.2.22) from (2.2.23), the following $T-2$ cross-linkage moment conditions are obtained:

$$E[(1/\mu_{it})(((\mu_{i,t-1}/\mu_{it})u_{it}-u_{i,t-1})u_{it}-(\mu_{i,t-1}/\mu_{it})y_{it})]=0 \quad \text{, for } t=3,\ldots,T \quad . \tag{2.2.24}$$

Eventually, a condensed full set of the moment conditions for the case where the assumption with respect to the equidispersion is imposed in addition to the implicit standard assumptions associated with predetermined explanatory variables is composed of (2.2.11), (2.2.19), (2.2.24) and (2.2.12). That is, under the assumption (2.2.1) with (2.2.6), the condensed full set is composed of the moment conditions (2.2.11), (2.2.19), (2.2.24) and (2.2.12). The moment conditions (2.2.11), (2.2.19) and (2.2.12) are linear with respect to γ , while (2.2.24) nonlinear.

Incidentally, another condensed full set of the moment conditions is constructed, mainly using the moment conditions proposed by Kitazawa (2007). For the case of predetermined explanatory variables, Kitazawa (2007) proposes the following $T-3$ moment conditions in addition to (2.2.11) and (2.2.12):

$$E[(1/\mu_{it})((\mu_{i,t-2}/\mu_{i,t-1})u_{i,t-1}-u_{i,t-2})u_{it}]=0 \quad , \quad \text{for } t=4,\ldots,T \quad , \tag{2.2.25}$$

which represent the relationships between $u_{i,t-2}u_{it}$ and $u_{i,t-1}u_{it}$ for $t=4,\ldots,T$. Further, for the case of predetermined explanatory variables, Kitazawa (2007) proposes the following $T-2$ moment conditions associated with the equidispersion:

$$E[(\mu_{i,t-1}^2/\mu_{it}^2)(u_{it}^2-y_{it})-(u_{i,t-1}^2-y_{i,t-1})]=0 \quad , \quad \text{for } t=3,\ldots,T \quad , \tag{2.2.26}$$

which represent the relationships between $u_{i,t-1}^2$ and u_{it}^2 for $t=3,\ldots,T$. Using any one in the cross-linkage moment conditions (2.2.19) and (2.2.24) in addition to the moment conditions (2.2.11), (2.2.25), (2.2.26) and (2.2.12), a condensed full set of the moment conditions is able to be constructed under the assumption (2.2.1) with (2.2.6) . However, in this case, the $T-3$ or $T-2$ moment conditions nonlinear with respect to γ increase, compared to the condensed full set described in the last paragraph.

2.3. Case of strictly exogenous explanatory variables

In this case, the assumption on the disturbance v_{it} is

$$E[v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad , \tag{2.3.1}$$

which is referred to as the original assumption for the case of strictly exogenous explanatory variables in this paper. Kitazawa (2007) constructs the implicit standard assumptions from (2.3.1) as follows:

$$E[y_{i1} v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad , \tag{2.3.2}$$

$$E[v_{is} v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad , \quad \text{for} \quad 2 \le s \le t-1 \quad , \tag{2.3.3}$$

$$E[x_{is} v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad , \quad \text{for} \quad 1 \le s \le T \quad , \tag{2.3.4}$$

$$E[\eta_i v_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad . \tag{2.3.5}$$

Here, the following assumption with respect to the equidispersion is assumed in addition to the implicit standard assumptions (2.3.2) – (2.3.5):

$$E[(v_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad . \tag{2.3.6}$$

Using the assumptions (2.3.2) – (2.3.6), new types of moment conditions based on the relationships holding between variances and covariances in the disturbances are proposed in addition to the quasi-differenced moment conditions reformed by Kitazawa (2007) for the case of strictly exogenous explanatory variables in this section. The line of constructing the moment conditions in manner of Kitazawa (2007) is sketched as below.

According to Kitazawa (2007), the observable analogues for (2.3.2), (2.3.3), (2.3.4), and (2.3.6) are obtained by replacing the unobservable variables v_{it} with the observable variables u_{it} :

$$E[y_{i1} u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=y_{i1} \phi_i \mu_{it} \quad , \tag{2.3.7}$$

$$E[u_{is} u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=\phi_i^2 \mu_{is} \mu_{it} + v_{is} \phi_i \mu_{it} \quad , \qquad \text{for} \quad 2 \le s \le t-1 \quad , \tag{2.3.8}$$

$$E[x_{is} u_{it} \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=x_{is} \phi_i \mu_{it} \quad , \qquad \text{for} \quad 1 \le s \le T \quad , \tag{2.3.9}$$

$$E[(u_{it}^2 - y_{it}) \mid y_{i1}, \eta_i, v_i^{t-1}, x_i^T]=\phi_i^2 \mu_{it}^2 \quad , \tag{2.3.10}$$

respectively.

Using the observable analogues (2.3.7) – (2.3.10), the relationships holding among $y_{i1} u_{it}$ for $t=2,\ldots,T$, among $u_{is} u_{it}$ for $s=2,\ldots,t$ and $t=2,\ldots,T$ and among $x_{is} u_{it}$ for $s=1,\ldots,T$ and $t=2,\ldots,T$ are solved through the intermediary of the unconditional expectation operator after weighting them with appropriate transformations of explanatory variables x_{it} for $t=1,\ldots,T$. With the intention of ruling out the redundancies of the moment conditions to be constructed, condensed full sets of the relationships above are used to construct a lean set of the moment conditions.

A condensed full set of the relationships among $y_{i1}u_{it}$ for $t=2,\dots,T$ is composed of the relationships between $y_{i1}u_{i,t-1}$ and $y_{i1}u_{it}$ for $t=3,\dots,T$. The other relationships among $y_{i1}u_{it}$ for $t=2,\dots,T$ are indirectly traced based on the trunk connections by exploiting the condensed full set.

In addition, a condensed full set of the relationships among $u_{is}u_{it}$ for $s=2,\dots,t$ and $t=2,\dots,T$ is composed of the relationships between $u_{is}u_{i,t-1}$ and $u_{is}u_{it}$ for $s=2,\dots,t-1$ and $t=3,\dots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\dots,T$. The other relationships among $u_{is}u_{it}$ for $s=2,\dots,t$ and $t=2,\dots,T$ are indirectly traced based on the trunk connections by exploiting the condensed full set.

Both relationships between $y_{i1}u_{i,t-1}$ and $y_{i1}u_{it}$ for $t=3,\dots,T$ and between $u_{is}u_{i,t-1}$ and $u_{is}u_{it}$ for $s=2,\dots,t-2$ and $t=4,\dots,T$ are solved by Kitazawa (2007) in order to construct the $(T-2)(T-1)/2$ moment conditions based on the product of the (observable) instrument variables and the quasi-differenced transformation reformed by Kitazawa (2007) for the case of strictly exogenous explanatory variables. That is,

$$E[y_{is}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})]=0 \quad , \qquad \text{for} \quad s=1,\dots,t-2 \ ; \ t=3,\dots,T \ , \qquad (2.3.11)$$

which hold even if the assumption (2.3.6) is not imposed.

Accordingly, in this paper, the relationships between $u_{i,t-1}^2$ and $u_{i,t-1}u_{it}$ for $t=3,\dots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\dots,T$ are solved in order to construct the moment conditions valid for the case of strictly exogenous explanatory variables. These moment conditions are also referred to as the cross-linkage moment conditions for the case of strictly exogenous explanatory variables in this paper, because they represent the relationships bridging between the observable analogues of variances of v_{it} and the observable analogues of covariances of v_{it} in the case of the equidispersion.

Further, the $(T-2)T$ moment conditions based on a condensed full set of the relationships among $x_{is}u_{it}$ for $s=1,\dots,T$ and $t=2,\dots,T$ are exhibited in Kitazawa (2007). They are

$$E[x_{is}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})]=0 \quad , \qquad \text{for} \quad s=1,\dots,T \ ; \ t=3,\dots,T \ , \qquad (2.3.12)$$

which are obtained by solving the relationships between $x_{is}u_{i,t-1}$ and $x_{is}u_{it}$ for $s=1,\dots,T$ and $t=3,\dots,T$. These moment conditions are also those based on the product of the (observable) instrument variables and the quasi-differenced transformation reformed by Kitazawa (2007) for the case of strictly exogenous explanatory variables.

From now on, two types of the cross-linkage moment conditions for the case of strictly exogenous explanatory variables are solved by using the relationships between $u_{i,t-1}^2$ and $u_{i,t-1}u_{it}$ for $t=3,\dots,T$ and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\dots,T$.

First, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{i,t-1}^2$ (weighted with $\mu_{it}/\mu_{i,t-1}$) and $u_{i,t-1}u_{it}$. Multiplying both sides of (2.3.10) dated $t-1$ by $\mu_{it}/\mu_{i,t-1}$ gives

$$E[(\mu_{it}/\mu_{i,t-1})(u_{i,t-1}^2-y_{i,t-1})\,|\,y_{i1},\eta_i,v_i^{t-2},x_i^T]=\phi_i^2\mu_{it}\mu_{i,t-1} \quad . \tag{2.3.13}$$

Applying the law of total expectation to (2.3.13) and (2.3.8) for $s=t-1$, it follows that

$$E[(\mu_{it}/\mu_{i,t-1})(u_{i,t-1}^2-y_{i,t-1})]=E[\phi_i^2\mu_{it}\mu_{i,t-1}] \quad , \tag{2.3.14}$$

$$E[u_{i,t-1}u_{it}]=E[\phi_i^2\mu_{i,t-1}\mu_{it}] \quad . \tag{2.3.15}$$

Subtracting (2.3.14) from (2.3.15) gives

$$E[u_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})+(\mu_{it}/\mu_{i,t-1})y_{i,t-1}]=0 \quad . \tag{2.3.16}$$

At this stage, creating the recursive equations

$$\begin{aligned}&E[y_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})]\\&=\gamma E[y_{i,t-2}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})]+E[u_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})] \quad ,\end{aligned} \tag{2.3.17}$$

from (2.1.2) dated $t-1$ and applying the moment conditions (2.3.11) for $s=t-2$, it can be seen that the following relationships hold:

$$E[u_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})]=E[y_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})] \quad , \tag{2.3.18}$$

Accordingly, plugging (2.3.18) into (2.3.16) gives the following $T-2$ cross-linkage moment conditions:

$$E[y_{i,t-1}(u_{it}-(\mu_{it}/\mu_{i,t-1})(u_{i,t-1}-1))]=0 \quad , \quad \text{for} \quad t=3,\ldots,T \quad , \tag{2.3.19}$$

in which the order reduction with respect to γ is realized, compared to (2.3.16).

Next, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{i,t-1}u_{it}$ (weighted with $\mu_{it}/\mu_{i,t-1}$) and u_{it}^2 . Multiplying (2.3.8) for $s=t-1$ by $\mu_{it}/\mu_{i,t-1}$ gives

$$E[(\mu_{it}/\mu_{i,t-1})u_{i,t-1}u_{it}\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^T]=\phi_i^2\mu_{it}^2+v_{i,t-1}\phi_i(\mu_{it}^2/\mu_{i,t-1}) \quad . \tag{2.3.20}$$

Applying the law of total expectation to (2.3.20) and (2.3.10),

$$E[(\mu_{it}/\mu_{i,t-1})u_{i,t-1}u_{it}]=E[\phi_i^2\mu_{it}^2] \quad , \tag{2.3.21}$$

$$E[(u_{it}^2-y_{it})]=E[\phi_i^2\mu_{it}^2] \quad . \tag{2.3.22}$$

Subtracting (2.3.21) from (2.3.22), the following $T-2$ cross-linkage moment conditions are obtained:

$$E[(u_{it}-(\mu_{it}/\mu_{i,t-1})u_{i,t-1})u_{it}-y_{it}]=0 \quad , \quad \text{for} \quad t=3,\ldots,T \quad . \tag{2.3.23}$$

Eventually, a condensed full set of the moment conditions for the case where the assumption with respect to the equidispersion is imposed in addition to the implicit standard assumptions associated with strictly exogenous explanatory variables is composed of (2.3.11), (2.3.19), (2.3.23) and (2.3.12). That is, under the assumption (2.3.1) with (2.3.6), the condensed full set is composed of the moment conditions (2.3.11), (2.3.19), (2.3.23) and (2.3.12). The moment conditions (2.3.11), (2.3.19) and (2.3.12) are linear with respect to γ, while (2.3.23) nonlinear.

Incidentally, another condensed full set of the moment conditions is constructed, mainly using the moment conditions proposed by Kitazawa (2007). For the case of strictly exogenous explanatory variables, Kitazawa (2007) proposes the following $T-3$ moment conditions in addition to (2.3.11) and (2.3.12):

$$E[(u_{i,t-1}-(\mu_{i,t-1}/\mu_{i,t-2})u_{i,t-2})u_{it}]=0 , \qquad \text{for } t=4,\ldots,T , \tag{2.3.24}$$

which represent the relationships between $u_{i,t-2}u_{it}$ and $u_{i,t-1}u_{it}$ for $t=4,\ldots,T$. The moment conditions (2.3.24) are able to be regarded as being conceptually equivalent to those proposed by Crépon and Duguet (1997), although the former is reformed compared to the latter allowing for the fact that the strictly exogenous explanatory variables are assumed. Further, for the case of strictly exogenous explanatory variables, Kitazawa (2007) proposes the following $T-2$ moment conditions associated with the equidispersion:

$$E[(u_{it}^2-y_{it})-(\mu_{it}^2/\mu_{i,t-1}^2)(u_{i,t-1}^2-y_{i,t-1})]=0 , \qquad \text{for } t=3,\ldots,T , \tag{2.3.25}$$

which represent the relationships between $u_{i,t-1}^2$ and u_{it}^2 for $t=3,\ldots,T$. Using any one in the cross-linkage moment conditions (2.3.19) and (2.3.23) in addition to the moment conditions (2.3.11), (2.3.24), (2.3.25) and (2.3.12), a condensed full set of the moment conditions is able to be constructed under the assumption (2.3.1) with (2.3.6). However, in this case, the $T-3$ or $T-2$ moment conditions nonlinear with respect to γ increase, compared to the condensed full set described in the last paragraph.

2.4. Case of mean-stationary dependent variables

In this case, the stationarity of the dependent and explanatory variables are additionally assumed for the case of predetermined explanatory variables in the LFM (2.1.1) (see Kitazawa, 2007).

When

$$E[\exp(k\,x_{it})\,|\,\eta_i]=E[\varphi_i(k)\,|\,\eta_i] , \qquad \text{for } t=1,\ldots,T \tag{2.4.1}$$

with k being any real number and

$$y_{i1}=(1/(1-\gamma))\phi_i\mu_{i1}+v_{i1} \tag{2.4.2}$$

with

$$E[v_{i1}\,|\,\eta_i,x_{i1}]=0 , \tag{2.4.3}$$

the dependent variables in the LFM are mean-stationary:

$$E[y_{it}]=(1/(1-\gamma))E[\phi_i\varphi_i(\beta)] \quad , \qquad \text{for} \quad t=1,\ldots,T \quad . \tag{2.4.4}$$

In this case, the observable analogue (2.2.7) is rewritten as

$$E[y_{i1}u_{it}\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=(1/(1-\gamma))\phi_i^2\mu_{i1}\mu_{it}+v_{i1}\phi_i\mu_{it} \quad . \tag{2.4.5}$$

Using the observable analogues (2.4.5) and (2.2.8) with (2.4.1), the relationships between $y_{i1}u_{i3}$ and $u_{i2}u_{i3}$ and between $u_{i,t-2}u_{it}$ and $u_{i,t-1}u_{it}$ for $t=4,\ldots,T$ through the intermediary of the unconditional expectation operator after weighting them with appropriate transformations of explanatory variables x_{it} for $t=1,\ldots,T$ are realized by Kitazawa (2007) as the following $T-2$ stationarity moment conditions for the case without requiring the assumption with respect to the equidispersion (2.2.6):

$$E[\Delta y_{i,t-1}(1/\mu_{it})u_{it}]=0 \quad , \qquad \text{for} \quad t=3,\ldots,T \quad , \tag{2.4.6}$$

where Δ is the first-differencing operator.

In addition, the relationships between $x_{i,t-1}u_{it}$ and $x_{it}u_{it}$ for $t=2,\ldots,T$ are also realized by Kitazawa (2007) as the following $T-1$ stationarity moment conditions for the case without requiring the assumption with respect to the equidispersion (2.2.6):

$$E[\Delta x_{it}(1/\mu_{it})u_{it}]=0 \quad , \qquad \text{for} \quad t=2,\ldots,T \quad . \tag{2.4.7}$$

From now on, the cross-linkage moment conditions for the case of the mean-stationary dependent variables are constructed in the situation where the assumptions (2.4.1) and (2.4.2) with (2.4.3) are imposed in addition to the assumption (2.2.1) with (2.2.6). They are solved by using the relationship between $y_{i1}u_{i2}$ and u_{i2}^2 and the relationships between $u_{i,t-1}u_{it}$ and u_{it}^2 for $t=3,\ldots,T$.

First, the relationship through the intermediary of the unconditional expectation operator is solved between $y_{i1}u_{i2}$ (weighted with $1/\mu_{i2}$) and u_{i2}^2 (weighted with $1/\mu_{i2}$). Multiplying (2.4.5) by $1/\mu_{it}$ gives

$$E[y_{i1}(1/\mu_{it})u_{it}\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=(1/(1-\gamma))\phi_i^2\mu_{i1}+v_{i1}\phi_i \quad . \tag{2.4.8}$$

In addition, multiplying (2.2.10) by $1/\mu_{it}$ gives

$$E[(1/\mu_{it})(u_{it}^2-y_{it})\,|\,y_{i1},\eta_i,v_i^{t-1},x_i^t]=\phi_i^2\mu_{it} \quad . \tag{2.4.9}$$

Applying law of total expectation to (2.4.8) and (2.4.9) and allowing for (2.4.1), it follows that

$$E[y_{i1}(1/\mu_{it})u_{it}]=(1/(1-\gamma))E[\phi_i^2\varphi_i(\beta)] \quad , \tag{2.4.10}$$

$$E[(1/\mu_{it})(u_{it}^2-y_{it})]=E[\phi_i^2\varphi_i(\beta)] \quad . \tag{2.4.11}$$

Subtracting (2.4.10) for $t=2$ multiplied by $1-\gamma$ from (2.4.11) for $t=2$ gives

$$E[(1/\mu_{i2})(\Delta y_{i2} u_{i2} - y_{i2})] = 0 \quad . \tag{2.4.12}$$

Next, the relationship through the intermediary of the unconditional expectation operator is solved between $u_{i,t-1} u_{it}$ (weighted with $1/\mu_{it}$) and u_{it}^2 (weighted with $1/\mu_{it}$). Allowing for (2.4.1), equation (2.2.22) is written as

$$E[u_{i,t-1}(1/\mu_{it}) u_{it}] = E[\phi_i^2 \varphi_i(\beta)] \quad . \tag{2.4.13}$$

Subtracting (2.4.13) from (2.4.11) gives

$$E[\Delta u_{it}(1/\mu_{it}) u_{it} - y_{it}(1/\mu_{it})] = 0 \quad . \tag{2.4.14}$$

Creating the recursive equation

$$E[\Delta y_{it}(1/\mu_{it}) u_{it}] = \gamma E[\Delta y_{i,t-1}(1/\mu_{it}) u_{it}] + E[\Delta u_{it}(1/\mu_{it}) u_{it}] \quad \text{, for} \quad t = 3, \ldots, T \quad , \tag{2.4.15}$$

from the first-differences of (2.1.2) and applying the moment conditions (2.4.6), it can be seen that the following relationships hold:

$$E[\Delta u_{it}(1/\mu_{it}) u_{it}] = E[\Delta y_{it}(1/\mu_{it}) u_{it}] \quad , \qquad \text{for} \quad t = 3, \ldots, T \quad . \tag{2.4.16}$$

Accordingly, plugging (2.4.16) into (2.4.14) gives the following $T-2$ moment conditions:

$$E[(1/\mu_{it})(\Delta y_{it} u_{it} - y_{it})] = 0 \quad , \qquad \text{for} \quad t = 3, \ldots, T \quad , \tag{2.4.17}$$

in which the order reduction with respect to γ is realized, compared to (2.4.14).

Writing (2.4.12) and (2.4.17) jointly, it follows that

$$E[(1/\mu_{it})(\Delta y_{it} u_{it} - y_{it})] = 0 \quad , \qquad \text{for} \quad t = 2, \ldots, T \quad , \tag{2.4.18}$$

which are referred to as the cross-linkage moment conditions for the case of stationary dependent variables (or the stationarity moment conditions for the case of the equidispersion) in this paper and whose number is $T-1$.

Eventually, a condensed full set of the moment conditions for the case of stationary dependent variables when the assumption with respect to the equidispersion is imposed in addition to the implicit standard assumptions associated with predetermined explanatory variables is composed of (2.2.11), (2.2.19), (2.4.18), (2.2.12) and (2.4.7). That is, under the assumptions (2.2.1) with (2.2.6), (2.4.1) and (2.4.2) with (2.4.3), the condensed full set is composed of the moment conditions (2.2.11), (2.2.19), (2.4.18), (2.2.12) and (2.4.7), all of which are linear with respect to γ .

Incidentally, another condensed full set of the moment conditions is constructed, mainly using the moment conditions proposed by Kitazawa (2007). Kitazawa (2007) proposes the following $T-2$ intertemporal homoscedasticity moment conditions associated with the equidispersion and stationary dependent variables for the case of predetermined explanatory variables:

$$E[u_{it}^2 - u_{i,t-1}^2] = 0 \quad , \qquad \text{for} \quad t = 3, \ldots, T \quad , \tag{2.4.19}$$

which imply that if both the equidispersion and the mean-stationarity hold with respect to the dependent variables for count panel data, the disturbances are homoscedastic over time. Using any one of the cross-linkage moment conditions (2.2.19) and (2.4.18) in addition to the moment conditions (2.2.11), (2.4.6), (2.4.19), (2.2.12) and (2.4.7), a condensed full set of the moment conditions is able to be constructed under the assumptions (2.2.1) with (2.2.6), (2.4.1) and (2.4.2) with (2.4.3). However, the intertemporal homoscedasticity moment conditions (2.4.19) are nonlinear with respect to γ .

2.5. Discussion

There can be a case where a manipulation is needed, when using any of the moment conditions (2.2.24) and (2.4.18) for the estimation of γ and β as well as when using (2.2.25), (2.4.6) and (2.4.7). If all values in x_{it} are positive (which is the ordinary in the econometric analysis), the estimates of β using these moment conditions seem to be in danger of going to infinity. In this case, in order that x_{it} contains both positive and negative values evenly, x_{it} needs to be transformed in deviation from an appropriate value b . That is, $\tilde{x}_{it}$ needs to be used in the estimations instead of x_{it} , where $\tilde{x}_{it}=x_{it}-b$. The selection of b by Windmeijer (2000) is the overall mean of x_{it} (i.e. $b=(1/(N\,T))\sum_{i=1}^{N}\sum_{t=1}^{T}x_{it}$).

2.6. GMM estimators

Any set of the moment conditions for the LFM (2.1.1) can be collectively written in the following $m\times 1$ vector form:

$$E[g_i(\theta)]=0 \quad , \tag{2.6.1}$$

where m is number of moment conditions, $\theta=[\gamma\ \beta]'$, $g_i(\theta)$ (which is the function of θ) is composed of the observable variables and θ for the individual i . Using the following empirical counterpart for (2.6.1):

$$\bar{g}(\theta) \ = \ (1/N)\sum_{i=1}^{N}g_i(\theta) \quad , \tag{2.6.2}$$

the GMM estimator $\hat{\theta}$ is constructed by minimizing the following criterion function with respect to θ :

$$\bar{g}(\theta)'\ W_N(\hat{\theta}_1)\ \bar{g}(\theta) \quad , \tag{2.6.3}$$

where the $m\times m$ optimal weighting matrix is given as follows by using a initial consistent estimator of θ (i.e. $\hat{\theta}_1$):

$$W_N(\hat{\theta}_1)=\left((1/N)\sum_{i=1}^{N}g_i(\hat{\theta}_1)g_i(\hat{\theta}_1)'\right)^{-1} \quad . \tag{2.6.4}$$

The efficient asymptotic variance of $\hat{\theta}$ is estimated by using

$$\hat{V}(\hat{\theta})=(1/N)\left(D(\hat{\theta})'\ W_N(\hat{\theta}_1)\ D(\hat{\theta})\right)^{-1} \quad , \tag{2.6.5}$$

where $D(\hat{\theta})=\partial \bar{g}(\theta)/\partial \theta'|_{\theta=\hat{\theta}}$.[3] The GMM estimations for the LFM are explained in detail in Windmeijer (2002, 2008).

For the case of predetermined explanatory variables, the following four GMM estimators are constructed: the GMM(qd) estimator using the moment conditions (2.2.11) and (2.2.12), the GMM(qdc) estimator using the moment conditions (2.2.11), (2.2.12) and (2.2.19), the GMM(pr) estimator using the moment conditions (2.2.11), (2.2.12) and (2.2.25) and the GMM(prc) estimator using the moment conditions (2.2.11), (2.2.12), (2.2.19) and (2.2.24).

For the case of strictly exogenous explanatory variables, the following four GMM estimators are constructed: the GMM(qe) estimator using the moment conditions (2.3.11) and (2.3.12), the GMM(qec) estimator using the moment conditions (2.3.11), (2.3.12) and (2.3.19), the GMM(ex) estimator using the moment conditions (2.3.11), (2.3.12) and (2.3.24) and the GMM(exc) estimator using the moment conditions (2.3.11), (2.3.12), (2.3.19) and (2.3.23).

For the case of mean-stationary dependent variables, the following two GMM estimators are constructed: the GMM(sa) estimator using the moment conditions (2.2.11), (2.2.12), (2.4.6) and (2.4.7) and GMM(sac) estimator using the moment conditions (2.2.11), (2.2.12), (2.4.7), (2.2.19) and (2.4.18).

It should be noted that the transformation described in previous subsection is needed to implement the GMM(pr), GMM(prc), GMM(sa) and GMM(sac) estimators.

3. Monte Carlo

In this section, some small sample performances of the GMM estimators exhibited in previous section are investigated with Monte Carlo experiments and the gains by the usage of the cross-linkage moment conditions are examined in small sample. The experiments are implemented by using an econometric software TSP version 4.5.[4]

3.1. Data generating process

The data generating process (DGP) is as follows:

$$y_{it} \sim Poisson(\gamma y_{i,t-1} + \exp(\beta x_{it} + \eta_i)) \quad , \tag{3.1.1}$$

$$y_{i,-TG+1} \sim Poisson((1/(1-\gamma))\exp(\beta x_{i,-TG+1} + \eta_i)) \quad , \tag{3.1.2}$$

$$x_{it} = \rho x_{i,t-1} + \tau \eta_i + \varepsilon_{it} \quad , \tag{3.1.3}$$

$$x_{i,-TG+1} = (1/(1-\rho))\tau \eta_i + (1/(1-\rho^2)^{(1/2)})\varepsilon_{i,-TG+1} \quad , \tag{3.1.4}$$

$$\eta_i \sim N(0, \sigma_\eta^2) \quad ; \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2) \quad ,$$

where $t=-TG+1,\ldots,-1,0,1,\ldots,T$ with TG being number of pre-sample periods to be generated. In the DGP, values are set to the parameters γ , β , ρ , τ , σ_η^2 and σ_ε^2 . The experiments are carried out with $TG=50$, the cross-sectional sizes $N=100$, 500 and 1000 , the numbers of periods used for the estimations $T=4$ and 8 , and the number of

3 It is conceivable that the usage of the finite sample corrected variance proposed by Windmeijer (2005, 2008) would be preferable in small sample.

4 See Hall and Cummins (2006) as for the details of the software.

replications $NR = 1000$.

The DGP setting is the same as that of Blundell et al. (2007), except for the initial condition of y_{it} . That is, the initial condition (3.1.2) denotes that the initial conditions of dependent variables are stationary. The DGP is configured with the explanatory variables x_{it} being strictly exogenous.

3.2. Estimators for comparison

The following three estimators are used for comparison: the Level estimator, the within group (WG) mean scaling estimator and the pre-sample mean (PSM) estimator. The Level and WG estimators are inconsistent in the DGP settings above, where N and T are able to be regarded as being large and small respectively. On the contrary, the PSM estimator is consistent if the long history is used in constructing the pre-sample means of dependent variables. The details on these estimators are described in Blundell et al. (1999, 2002) and Kitazawa (2007).

3.3. Monte Carlo results

Results for the Monte Carlo experiments are exhibited in Table 1 for the situation of moderately persistent y_{it} and x_{it} when $T=4$, in Table 2 for the situation of moderately persistent y_{it} and x_{it} when $T=8$, in Table 3 for the situation of considerably persistent y_{it} and x_{it} when $T=8$ and in Table 4 for the situation of considerably persistent y_{it} and extremely persistent x_{it} when $T=8$. The settings of values of parameters are the same as those in Blundell et al. (2002) and for the case of strictly exogenous explanatory variables in Kitazawa (2007), except for the initial conditions of dependent variables. The bias and rmse are calculated for the parameters γ and β estimated by using the Level, WG, GMM and PSM estimators.

The endemic upward and downward biases are found for the Level and WG estimators respectively, while the PSM estimator behaves well as the pre-sample length used elongates.

The instruments used for the GMM estimators are curtailed so that the past dependent variables (y_{it}) dated $t-3$ and before are not used for the quasi-differenced equation dated t and further for the GMM(qd), GMM(qdc), GMM(pr), GMM(prc), GMM(sa) and GMM(sac) estimators the past explanatory variables (x_{it}) dated $t-3$ and before are not used for the quasi-differenced equation dated t . The size alleviation of bias and rmse for the all GMM estimators is found as N increases, which is the reflection of the consistency. Then, the results on the GMM estimators suggest that some gains and no loss seem to be obtained in small sample by using the cross-linkage moment conditions associated with the equidispersion.

Firstly, it is conceivable that the GMM estimators using the cross-linkage moment conditions outperform the conventional GMM(qd) estimator.

Secondly, comparing the results using the GMM(qd) estimator with those using the GMM(qdc) estimator (where both estimators are tailored to the specification for the case of predetermined explanatory variables) and comparing the results using the GMM(qe) estimator with those using the GMM(qec) estimator (where both estimators are tailored to the specification for the case of strictly exogenous explanatory variables), it can be said that the GMM estimators additionally incorporating parts of the cross-linkage moment conditions perform better than those without incorporating the cross-linkage moment conditions. This is conspicuous for the situation of moderately persistent y_{it} and x_{it} when $T=4$.

Thirdly, comparing the results using the GMM(pr) estimator with those using the GMM(prc) estimator (where both estimators are tailored to the specification for the case of predetermined explanatory variables), comparing the results using the GMM(ex) estimator with those using the GMM(exc) estimator (where both estimators are tailored to the specification for the case of strictly exogenous explanatory variables) and comparing the results using the GMM(sa) estimator with those using the GMM(sac) estimator (where both estimators are tailored to the specification for the case of mean-stationary dependent variables), it can be said that the GMM estimators utilizing the condensed full set incorporating the cross-linkage moment conditions maximally does not underperform those without incorporating the cross-linkage moment conditions.[5]

After all, these Monte Carlo experiments say that the usage of the cross-linkage moment conditions associated with the equidispersion improves or does not at least vitiate the small sample performances for the case of the equidispersion, as long as comparing the results of the GMM estimators incorporating the cross-linkage moment conditions with those incorporating no cross-linkage moment condition. It is particularly worth noting that the GMM estimators incorporating the cross-linkage moment conditions perform better than the conventional GMM(qd) estimator.

4. Conclusion

In this paper, the cross-linkage moment conditions associated with the equidispersion for count panel data model were proposed for the case of predetermined explanatory variables, for the case of strictly exogenous explanatory variables and for the case of mean-stationary dependent variables. In the Monte Carlo experiments whose DGP are of the Poisson model, it was shown that the GMM estimators incorporating the cross-linkage moment conditions behave better than the conventional GMM(qd) estimator in small sample and some gains are found when using the cross-linkage moment conditions.

References

Ahn, S.C. (1990). Three essays on share contracts, labor supply, and the estimation of models for dynamic panel data. *Unpublished Ph.D. Dissertation (Michigan State University, East Lansing, MI)*.

Ahn, S.C. & Schmidt, P. (1995). Efficient estimation of models for dynamic panel data. *Journal of Econometrics* **68**, 5-27.

Blundell, R., Griffith, R. & Van Reenen, J. (1999). Market share, market value and innovation in a panel of British manufacturing firms. *Review of Economic Studies* **66**, 529-554.

Blundell, R., Griffith, R. & Windmeijer, F. (2002). Individual effects and dynamics in count data models. *Journal of Econometrics* **108**, 113-131.

Chamberlain, G. (1992). Comment: sequential moment restrictions in panel data. *Journal of Business and Economic Statistics* **10**, 20-26.

Crépon, B. & Duguet, E. (1997). Estimating the innovation function from patent numbers: GMM on count panel data. *Journal of Applied Econometrics* **12**, 243-263.

Hall, B.H. & Cummins, C. (2006). *TSP 5.0 user's guide*, TSP international.

Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica* **50**, 1029-1054.

5 The condensed full sets are constructed on the basis of the curtailed sets mentioned above.

Hausman, J. A., Hall, B. H. & Griliches, Z. (1984). Econometric models for count data with an application to the patent-R&D relationship. *Econometrica* **52**, 909-938.

Kitazawa, Y. (2007). Some additional moment conditions for a dynamic count panel data model. *Kyushu Sangyo University, Faculty of Economics, Discussion Paper*, No. **29**.

Lancaster, T. (2002). Orthogonal parameters and panel data. *Review of Economic Studies* **69**, 647-666.

Windmeijer, F. (2000). Moment conditions for fixed effects count data models with endogenous regressors. *Economics Letters* **68**, 21-24.

Windmeijer, F. (2002). ExpEnd, A Gauss programme for non-linear GMM estimation of ***Exp***onential models with ***End***ogenous regressors for cross section and panel data. *The Institute for Fiscal Studies, Department of Economics, UCL, Cemmap Working Paper*, **CWP 14/02**.

Windmeijer, F. (2005). A finite sample correction for the variance of linear efficient two-step GMM estimators. *Journal of Econometrics* **126**, 25-51.

Windmeijer, F. (2008). GMM for panel count data models. *Laszlo, M. and P. Sevestre (eds.), The econometrics of panel data. Fundamentals and recent developments in theory*, Springer.

Wooldridge, J. M. (1997). Multiplicative panel data models without the strict exogeneity assumption. *Econometric Theory* **13**, 667-678.

Table 1

Monte Carlo results for LFM, T=4

(Situation of moderately persistent y_{it} and x_{it})

$\gamma=0.5$; $\beta=0.5$; $\rho=0.5$; $\tau=0.1$; $\sigma_{\eta}^{2}=0.5$; $\sigma_{\varepsilon}^{2}=0.5$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.256	0.264	0.273	0.275	0.278	0.279
	β	0.545	0.656	0.549	0.571	0.557	0.573
WG	γ	**−0.452**	**0.463**	**−0.446**	**0.449**	**−0.446**	**0.447**
	β	**−0.260**	**0.272**	**−0.261**	**0.263**	**−0.263**	**0.264**
GMM(qd)	γ	−0.274	0.398	−0.104	0.161	−0.061	0.112
	β	−0.259	0.371	−0.124	0.219	−0.078	0.172
GMM(qdc)	γ	**−0.054**	**0.155**	**−0.006**	**0.066**	**−0.001**	**0.045**
	β	**−0.134**	**0.288**	**−0.028**	**0.148**	**−0.013**	**0.104**
GMM(pr)	γ	−0.090	0.214	−0.037	0.092	−0.016	0.067
	β	−0.159	0.282	−0.059	0.157	−0.026	0.134
GMM(prc)	γ	**−0.024**	**0.158**	**−0.007**	**0.062**	**−0.002**	**0.043**
	β	**−0.166**	**0.281**	**−0.056**	**0.147**	**−0.029**	**0.107**
GMM(qe)	γ	−0.222	0.306	−0.060	0.110	−0.031	0.075
	β	−0.148	0.220	−0.049	0.111	−0.025	0.086
GMM(qec)	γ	**−0.074**	**0.156**	**−0.004**	**0.060**	**0.000**	**0.042**
	β	**−0.083**	**0.185**	**−0.008**	**0.088**	**−0.001**	**0.067**
GMM(ex)	γ	−0.103	0.226	−0.035	0.092	−0.015	0.058
	β	−0.106	0.214	−0.032	0.106	−0.013	0.077
GMM(exc)	γ	**−0.008**	**0.170**	**0.007**	**0.063**	**0.006**	**0.042**
	β	**−0.073**	**0.233**	**−0.005**	**0.094**	**0.001**	**0.069**
GMM(sa)	γ	−0.023	0.139	−0.019	0.079	−0.010	0.059
	β	−0.053	0.212	−0.023	0.137	−0.012	0.104
GMM(sac)	γ	**0.038**	**0.115**	**0.016**	**0.060**	**0.011**	**0.043**
	β	**−0.022**	**0.215**	**0.016**	**0.133**	**0.012**	**0.099**
PSM	γ(4)	0.132	0.156	0.157	0.162	0.163	0.167
	β(4)	0.191	0.296	0.205	0.225	0.211	0.229
	γ(8)	**0.104**	**0.132**	**0.125**	**0.131**	**0.130**	**0.135**
	β(8)	**0.141**	**0.228**	**0.148**	**0.165**	**0.152**	**0.165**
	γ(25)	0.046	0.091	0.061	0.072	0.066	0.073
	β(25)	0.058	0.139	0.062	0.083	0.065	0.078
	γ(50)	**0.020**	**0.081**	**0.033**	**0.050**	**0.038**	**0.048**
	β(50)	**0.031**	**0.119**	**0.032**	**0.059**	**0.035**	**0.052**

Notes: (1) The number of replications is 1000. (2) The instrument sets for GMM estimators include no time dummies. (3) As for the PSM estimator, the figures in the parentheses next to γ and β imply numbers of the latest pre-sample periods used for the estimations. (4) Both of the replications where no convergence is achieved in the estimations and/or where the estimates of γ and β whose absolute values exceed 10 are obtained (in the Level and PSM estimators) are eliminated when calculating the values of the Monte Carlo statistics. Their rates are below about three percent in total for each experiment. (5) The individuals where the pre-sample means are zero are eliminated in each replication when estimating the parameters of interest using the PSM estimator. (6) The values of the Monte Carlo statistics exhibited in the table are those obtained using the true values of γ and β as the starting values in the optimizations for each replication. The values of the statistics obtained using the true values are not much different from those obtained using two different types of the starting values. The differences are below about 0.01 in terms of the absolute value in nearly all cases and below about 0.02 in almost all cases.

Table 2

Monte Carlo results for LFM, T=8

(Situation of moderately persistent y_{it} and x_{it})

$\gamma=0.5$; $\beta=0.5$; $\rho=0.5$; $\tau=0.1$; $\sigma^2_\eta=0.5$; $\sigma^2_\varepsilon=0.5$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.262	0.267	0.275	0.277	0.278	0.279
	β	0.537	0.586	0.550	0.565	0.559	0.568
WG	γ	**−0.189**	**0.198**	**−0.184**	**0.186**	**−0.185**	**0.186**
	β	**−0.126**	**0.139**	**−0.127**	**0.130**	**−0.127**	**0.129**
GMM(qd)	γ	−0.229	0.261	−0.076	0.096	−0.046	0.062
	β	−0.232	0.265	−0.105	0.131	−0.066	0.091
GMM(qdc)	γ	**−0.147**	**0.185**	**−0.019**	**0.044**	**−0.007**	**0.027**
	β	**−0.217**	**0.257**	**−0.057**	**0.093**	**−0.024**	**0.057**
GMM(pr)	γ	−0.006	0.128	−0.029	0.054	−0.023	0.040
	β	−0.117	0.190	−0.064	0.096	−0.043	0.069
GMM(prc)	γ	**−0.007**	**0.103**	**−0.006**	**0.038**	**−0.003**	**0.026**
	β	**−0.154**	**0.213**	**−0.066**	**0.097**	**−0.031**	**0.060**
GMM(qe)	γ	−0.321	0.337	−0.080	0.092	−0.041	0.050
	β	−0.233	0.243	−0.081	0.091	−0.042	0.053
GMM(qec)	γ	**−0.261**	**0.281**	**−0.035**	**0.049**	**−0.012**	**0.025**
	β	**−0.221**	**0.236**	**−0.053**	**0.069**	**−0.021**	**0.038**
GMM(ex)	γ	0.011	0.179	−0.021	0.055	−0.019	0.036
	β	−0.129	0.212	−0.039	0.065	−0.025	0.045
GMM(exc)	γ	**−0.036**	**0.140**	**−0.008**	**0.042**	**−0.003**	**0.025**
	β	**−0.115**	**0.222**	**−0.038**	**0.075**	**−0.018**	**0.042**
GMM(sa)	γ	−0.012	0.079	−0.012	0.043	−0.009	0.031
	β	−0.070	0.134	−0.027	0.073	−0.017	0.053
GMM(sac)	γ	**0.029**	**0.077**	**0.010**	**0.037**	**0.007**	**0.027**
	β	**−0.012**	**0.142**	**−0.003**	**0.074**	**0.004**	**0.058**
PSM	γ(4)	0.145	0.155	0.162	0.165	0.165	0.167
	β(4)	0.197	0.231	0.210	0.222	0.216	0.221
	γ(8)	**0.115**	**0.127**	**0.131**	**0.135**	**0.134**	**0.136**
	β(8)	**0.145**	**0.178**	**0.155**	**0.164**	**0.160**	**0.165**
	γ(25)	0.054	0.075	0.068	0.073	0.070	0.073
	β(25)	0.063	0.100	0.068	0.078	0.071	0.076
	γ(50)	**0.027**	**0.059**	**0.039**	**0.047**	**0.040**	**0.044**
	β(50)	**0.033**	**0.078**	**0.036**	**0.049**	**0.039**	**0.045**

Notes: See Table 1.

Table 3

Monte Carlo results for LFM, T=8

(Situation of considerably persistent y_{it} and x_{it})

$\gamma=0.7$; $\beta=1$; $\rho=0.9$; $\tau=0$; $\sigma_\eta^2=0.5$; $\sigma_\varepsilon^2=0.05$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.169	0.173	0.181	0.182	0.181	0.182
	β	0.420	0.644	0.433	0.489	0.427	0.455
WG	$\boldsymbol{\gamma}$	**−0.250**	**0.257**	**−0.245**	**0.247**	**−0.245**	**0.245**
	$\boldsymbol{\beta}$	**−0.362**	**0.398**	**−0.365**	**0.372**	**−0.364**	**0.367**
GMM(qd)	γ	−0.359	0.413	−0.105	0.136	−0.061	0.086
	β	−0.682	0.889	−0.397	0.586	−0.278	0.432
GMM(qdc)	$\boldsymbol{\gamma}$	**−0.223**	**0.277**	**−0.019**	**0.049**	**−0.006**	**0.030**
	$\boldsymbol{\beta}$	**−0.659**	**0.879**	**−0.238**	**0.461**	**−0.116**	**0.297**
GMM(pr)	γ	0.035	0.149	−0.017	0.070	−0.018	0.048
	β	−0.344	0.601	−0.217	0.364	−0.162	0.290
GMM(prc)	$\boldsymbol{\gamma}$	**0.025**	**0.107**	**0.006**	**0.041**	**0.000**	**0.027**
	$\boldsymbol{\beta}$	**−0.364**	**0.605**	**−0.273**	**0.399**	**−0.180**	**0.293**
GMM(qe)	γ	−0.521	0.542	−0.140	0.156	−0.066	0.079
	β	−0.628	0.647	−0.292	0.321	−0.156	0.192
GMM(qec)	$\boldsymbol{\gamma}$	**−0.430**	**0.453**	**−0.058**	**0.074**	**−0.018**	**0.033**
	$\boldsymbol{\beta}$	**−0.618**	**0.652**	**−0.184**	**0.237**	**−0.063**	**0.131**
GMM(ex)	γ	0.069	0.193	*−0.004*	*0.073*	*−0.009*	*0.046*
	β	−0.412	0.809	*−0.091*	*0.227*	*−0.046*	*0.140*
GMM(exc)	$\boldsymbol{\gamma}$	**0.034**	**0.119**	**0.014**	**0.045**	**0.007**	**0.029**
	$\boldsymbol{\beta}$	**−0.258**	**0.859**	**−0.032**	**0.251**	**−0.001**	**0.140**
GMM(sa)	γ	0.000	0.080	−0.012	0.043	−0.011	0.033
	β	−0.275	0.448	−0.134	0.248	−0.094	0.201
GMM(sac)	$\boldsymbol{\gamma}$	**0.032**	**0.084**	**0.009**	**0.035**	**0.003**	**0.025**
	$\boldsymbol{\beta}$	**−0.220**	**0.460**	**−0.128**	**0.256**	**−0.075**	**0.201**
PSM	γ(4)	0.114	0.125	0.134	0.137	0.135	0.137
	β(4)	0.052	0.432	0.070	0.205	0.066	0.155
	$\boldsymbol{\gamma}$**(8)**	**0.104**	**0.115**	**0.123**	**0.126**	**0.124**	**0.126**
	$\boldsymbol{\beta}$**(8)**	**0.012**	**0.330**	**0.031**	**0.174**	**0.025**	**0.117**
	γ(25)	0.076	0.091	0.093	0.096	0.093	0.095
	β(25)	−0.007	0.219	0.002	0.109	−0.002	0.077
	$\boldsymbol{\gamma}$**(50)**	**0.055**	**0.073**	**0.069**	**0.074**	**0.069**	**0.072**
	$\boldsymbol{\beta}$**(50)**	**0.001**	**0.186**	**0.005**	**0.084**	**0.003**	**0.062**

Notes: See Table 1. Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are mildly different from those obtained using the two different values.

Table 4

Monte Carlo results for LFM, T=8

(Situation of considerably persistent y_{it} and extremely persistent x_{it})

$\gamma=0.7$; $\beta=1$; $\rho=0.95$; $\tau=0$; $\sigma_{\eta}^{2}=0.5$; $\sigma_{\varepsilon}^{2}=0.015$

		N=100		N=500		N=1000	
		bias	rmse	bias	rmse	bias	rmse
Level	γ	0.171	0.175	0.183	0.184	0.184	0.185
	β	0.241	0.489	0.240	0.318	0.249	0.288
WG	γ	**−0.276**	**0.282**	**−0.272**	**0.273**	**−0.270**	**0.271**
	β	**−0.354**	**0.450**	**−0.360**	**0.379**	**−0.357**	**0.368**
GMM(qd)	γ	−0.435	0.500	−0.142	0.189	−0.070	0.106
	β	−0.776	1.433	−0.561	1.109	−0.397	0.842
GMM(qdc)	γ	**−0.230**	**0.286**	**−0.014**	**0.048**	**0.000**	**0.030**
	β	**−0.769**	**1.350**	**−0.287**	**0.787**	**−0.194**	**0.599**
GMM(pr)	γ	0.041	0.159	0.001	0.081	−0.006	0.058
	β	−0.290	0.830	−0.173	0.532	−0.143	0.441
GMM(prc)	γ	**0.030**	**0.106**	**0.017**	**0.044**	**0.008**	**0.030**
	β	**−0.313**	**0.781**	**−0.302**	**0.564**	**−0.257**	**0.468**
GMM(qe)	γ	−0.599	0.621	−0.190	0.212	−0.090	0.107
	β	−0.635	0.678	−0.337	0.383	−0.187	0.247
GMM(qec)	γ	**−0.476**	**0.497**	**−0.060**	**0.079**	**−0.015**	**0.033**
	β	**−0.629**	**0.698**	**−0.188**	**0.295**	**−0.056**	**0.184**
GMM(ex)	γ	0.057	0.184	*0.005*	*0.087*	*−0.008*	*0.058*
	β	−0.373	1.179	*−0.073*	*0.346*	*−0.029*	*0.212*
GMM(exc)	γ	**0.026**	**0.115**	**0.020**	**0.047**	**0.011**	**0.031**
	β	**−0.267**	**1.373**	**0.001**	**0.366**	**0.007**	**0.207**
GMM(sa)	γ	0.017	0.088	0.003	0.046	−0.001	0.035
	β	−0.286	0.593	−0.139	0.362	−0.092	0.283
GMM(sac)	γ	**0.054**	**0.107**	**0.024**	**0.046**	**0.013**	**0.031**
	β	**−0.230**	**0.605**	**−0.145**	**0.361**	**−0.092**	**0.284**
PSM	$\gamma(4)$	0.108	0.119	0.128	0.131	0.131	0.132
	$\beta(4)$	−0.199	0.420	−0.194	0.252	−0.190	0.221
	$\gamma(8)$	**0.097**	**0.108**	**0.116**	**0.118**	**0.118**	**0.120**
	$\beta(8)$	**−0.242**	**0.395**	**−0.239**	**0.277**	**−0.234**	**0.253**
	$\gamma(25)$	0.070	0.084	0.086	0.090	0.089	0.091
	$\beta(25)$	−0.235	0.331	−0.233	0.258	−0.226	0.239
	$\gamma(50)$	**0.052**	**0.069**	**0.067**	**0.072**	**0.070**	**0.073**
	$\beta(50)$	**−0.170**	**0.270**	**−0.166**	**0.194**	**−0.162**	**0.175**

Notes: See Table 1. Further, (7) The values of the Monte Carlo statistics written in an italic type for the GMM estimators are obtained using one of the two different types of the starting values, whose differences from those obtained using another are below about 0.01 in terms of the absolute value. The reason why these values are exhibited in the table is that the values of the statistics obtained using the true values are mildly different from those obtained using the two different values.