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The global resource market and volatility

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Abstract

In this paper, we focus on the role of the global resource market in a dynamic model that incorporates a renewable resource sector. In particular, we show that resource-abundant countries become more volatile when the resource sector has access to the global market. By contrast, if the resource sector does not have access to its market, volatility is greatly reduced. To confirm that our main finding is robust, we incorporate into our basic framework endogenous labor-leisure choice and investment in the natural resource.

Key words: Global resource market; Renewable resources; Endogenous rate of time preference; Indeterminacy

JEL classification: F41; H21; Q22; Q28

1 Introduction

Since the late 1980s, much evidence has been presented to suggest that natural resource abundance is a curse for developing countries. More specifically, the evidence suggests that natural resource abundance increases the likelihood that a developing

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country will experience slow economic growth (see, e.g., the seminal work of Sachs and Warner, 1995). In opposition to this conventional interpretation of the resource curse, some researchers have found evidence of the importance of volatility for the resource curse. For instance, van der Ploeg and Poelhekke (2009, 2010) conclude that resource-rich countries experience low rates of growth through greater macroeconomic volatility. In particular, it is confirmed that the effect of natural resource wealth on growth is positive; however, its positive effect is dominated by the indirect negative effect of resource wealth on growth through volatility. Thus, they conclude that *volatility is a quintessential feature of the resource curse.*[1]

The most common causes of volatility are the preferences of inhabitants and the form of international participation.[2] First, the distinctive features of resource wealth may make resource benefits more or less attractive to a country's inhabitants. In other words, resource wealth undoubtedly influences whether a country's inhabitants decide to exploit its natural resources to obtain resource revenues; thus, resource wealth may tempt them to overharvest resources irrespective of future generations. That is, an abundance of natural resources might make a country's inhabitants myopic. Indeed, in the literature on resource-rich states, it is typically assumed that the possession of abundant resources engenders myopic decision making (e.g., Mahdavy, 1970; Kahl, 1997). This myopic decision making might be a cause of economic volatility.

Second, it is well known that markets for primary products are volatile (e.g., Ross, 1999; Davis and Tilton, 2005; van der Ploeg and Poelhekke, 2009). In particular, countries whose exports are dominated by a single resource commodity tend to face considerable volatility in export earnings. According to Davis and Tilton (2005), price variations of 30 percent or more within a year or two are common. This means that when the commodities produced by natural resources are used as an alternative

[1] See van der Ploeg (2011) for a survey of the resource curse.

[2] Other reasons include economic diversification, macroeconomic policies, natural resource funds, transparency and accountability and direct distribution to the general population.

source of export revenue, the volatility of resource-wealth states increases.[3] Moreover, empirical researchers in this field make use of data on primary-product exports to measure resource dependence and abundance, including Sachs and Warner (1995), implying that greater primary-product exports increase volatility. This evidence indicates that economies that participate in the global resource market tend to be more volatile.[4]

The objective of this paper is to examine whether a resource-dependent economy suffers from the indeterminacy problem, that is, whether there is a continuum of equilibrium paths that depend on households' beliefs when there is myopic decision making and participation in international markets. Such indeterminacy implies that resource-wealthy economies experience high volatility. Moreover, we are interested in how resource-abundant states might avoid the indeterminacy problem, and in particular, whether partial withdrawal from global markets might reduce volatility. We examine whether economic volatility is reduced when the nonnatural resource sector remains open following the withdrawal of the natural resource sector from the global market.

Our model has several features. First, we incorporate the natural resource sector into a dynamic model of a small open economy. This means that our model incorporates tradable and consumable commodities in the natural and the nonnatural resource sectors. In particular, as in Elíasson and Turnovsky (2004), renewable

[3] Analyzing Turkmenistan, Uzbekistan, Russia, Kazakhstan and Azerbaijan as independent states following the Soviet Union's collapse in 1991, Luong and Weinthal (2001) show that Kazakhstan and Azerbaijan, which sell their resource commodities domestically, managed to avoid the resource curse much better than Uzbekistan and Russia, which export their resource commodities. According to Luong and Weinthal (2001), Uzbekistan's cotton sector produced more than 65 percent of its gross output and employed approximately 40 percent of the labor force in the mid-1980s.

[4] Brunnschweiler and Bulte (2008) critically evaluate the use of natural resource exports as a percentage of GDP as the most widely used measure of resource abundance in the literature on the resource curse. They argue that it is at best an imperfect proxy because resource-wealthy countries that have also developed other industries may not be dependent on primary exports.

natural resources are represented by a forest, a fishery and cotton. Second, household preferences depend on the natural resource. In particular, households become myopic as natural resource wealth increases. Thus, the level of natural resources influences household preferences. On the other hand, individuals have no influence on aggregate natural resource wealth. That is, there is an environmental externality in time preference.

Our paper is closely related to those of Elíasson and Turnovsky (2004) and Yanase (2011).[5] Following Sachs and Warner's (1995, 2001) arguments on the resource curse, Elíasson and Turnovsky (2004) use a Romer-type small open-economy model, in which the equilibrium is defined as a well-behaved saddle path, to show that a renewable natural resource has an adverse effect on the growth rate. Although we are also interested in the resource curse, unlike Elíasson and Turnovsky (2004), we relate the resource curse to volatility.

Like Yanase (2011), we incorporate an environmental externality into the discount function. However, whereas Yanase (2011) allows the discount rate to depend on total pollution and private consumption, we assume that it depends on the aggregate level of renewable natural resources. Whereas Yanase's (2011) main objective is to show that the dynamic equilibrium is indeterminate, we are mainly interested in whether a small open economy can find a desirable stable path by withdrawing from the global resource market.

In Section 2, we develop a dynamic model incorporating a renewable resource that is used to produce an exported commodity, and then show what happens when the country withdraws from the global resource market. In Section 3, we extend our analysis to incorporate endogenous labor supply or investment in the natural resource. Section 4 concludes the paper.

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[5] Our paper in environmental economics is also related to standard neoclassical economic analyses in which there is no resource sector. For example, Bian and Meng (2004) and Meng (2006) show that the equilibrium path followed by a small open economy is indeterminate when there is an endogenous rate of time preference based on aggregate capital or consumption (rather than on natural resources, as in our model).

2 Participation in the global resource market

2.1 The basic framework

Let us consider a small open economy that can access the global resource market. The small economy has a continuum of identical and infinitely lived households, and the total population is constant over time. The economy is endowed with a stock of a renewable resource. The level of the renewable resource held by a household (all households) is N_t ($N_{\text{all},t}$).

Letting Z_t be the rate of harvesting an input for production, the evolution of the renewable resource held by a household is:

$$\dot{N}_t = G(N_t) - Z_t, \text{ where } G'(N_t) > (<) 0 \text{ if } N < (>) \bar{N} \text{ and } G''(N_t) < 0. \tag{1}$$

The function $G(N_t)$ represents the reproduction of the renewable resource and has an inverted U shape with $G(0) = G(N_{\max}) = 0$, where $N_{\max}$ is the carrying capacity of the natural resource, which is the level at which its growth ceases.[6] Because the function is strictly concave, there is a unique value $\bar{N}$ at which $G'(\bar{N}) = 0$, where $\bar{N}$ represents the level of the renewable resource that provides the maximum sustained yield. The dynamic movement of the natural resource in our basic framework is the same as in Elíasson and Turnovsky (2004) in the sense that the natural resource is characterized by harvesting and regeneration.

The economy has two sectors: a tradable natural resource sector and a tradable nonnatural resource sector. The commodity in the nonnatural resource sector is produced by physical capital K_t and labor l_t according to the neoclassical production function $F(K_t, l_t) = l_t f\left(\frac{K_t}{l_t}\right)$. For now, we assume that individuals are endowed with a unit of time that is inelastically supplied and that the firms in this sector produce either the consumption good C_t^y or the investment good I_t^y. In equilibrium, the firms in the nonnatural resource sector hire capital and labor to satisfy the following

[6] Brander and Scott-Taylor (1997), Ayong Le Kama (2001), Elíasson and Turnovsky (2004) and Fullerton and Kim (2008), among others, make use of the same form of reproduction function.

equalities:

$$R_t = f\left(k_t\right), \quad W_t = f\left(k_t\right) - k_t f'\left(k_t\right), \quad k_t = \frac{K_t}{l_t}, \tag{2}$$

where R_t and W_t are the competitive rental rate of capital and the wage rate, respectively. The commodity in the resource sector is produced by the harvested natural resource according to the production function $h(Z_t)$, with $h'(Z_t) > 0$, $h''(Z_t) < 0$, $\lim_{Z_t \to 0} h'(Z_t) > 0$ and $\lim_{Z_t \to \infty} h'(Z_t) = 0$. Under the assumption that domestic residents only have access to a risk-free bond B_t, the rate of return on which r is exogenously determined abroad, the budget constraint is:

$$\dot{B}_t = rB_t + W_t + R_t K_t + ph(Z_t) - C_t^y - pC_t^n - I_t^y, \tag{3}$$

where C_t^n represents consumption of the commodity produced in the resource sector and p is the relative world price when the price of C_t^y is unity. Because the commodity C_t^n is exported on the global resource market, the assumption of competitiveness implies that the relative world price is fixed.

The capital accumulation constraint is as follows:

$$\dot{K}_t = I_t^y - \delta K_t, \tag{4}$$

where δ is the constant rate of capital depreciation.

In our basic framework, each household seeks to maximize the present discounted value of its lifetime utility, U_B, as follows:

$$\begin{aligned} &U_B \equiv \int_0^{+\infty} [u(C_t^y) + v(N_t) + w(C_t^n)] \exp[-\Theta_t] dt, \\ &u'(C_t^y) > 0,\ u''(C_t^y) < 0,\ v'(N_t) > 0,\ v''(N_t) < 0,\ w'(C_t^n) > 0,\ \text{and}\ w''(C_t^n) < 0. \end{aligned} \tag{5}$$

In (5), both consumption and environmental amenities contribute to utility levels given by $u(C_t^y)$, $v(N_t)$ and $w(C_t^n)$, which satisfy Inada conditions. In particular, we assume that as the stock of the natural resources held by a household increases, the household obtains higher utility. Θ_t denotes the integral value of the instantaneous rate of time preference from the initial time to the current time.

We assume that the discount rate depends on the environmental input. Although this assumption is similar to those made by Lines (2005) and Yanase (2011), they assume that the discount rate is affected by pollution input, rather than environmental input. The household's discount factor is defined by:

$$\Theta_t = \int_0^t \rho(N_{\text{all},v})dv, \; \frac{d\Theta_t}{dt} = \rho(N_{\text{all},t}), \; \Theta_0 = 0, \; \rho'(N_{\text{all},t}) > 0, \; \text{and } \rho(0) \geq 0, \tag{6}$$

where $\rho(N_{\text{all},t})$ is the instantaneous rate of time preference, the evolution of which depends on the economy's aggregate level of the natural resource. Our interpretation of the assumption that the degree of impatience is positively related to the stock of the natural resource (i.e., $\rho' > 0$) is as follows. When the natural resource is abundant, individuals are myopic. That is, they do not care about conserving the natural resource for the future. However, as the stock of the natural resource diminishes, the rate of time preference falls such that households become more patient as they begin to worry about the exhaustion of the natural resource in the future.

Taking account of the maximization problem of the representative household, we assume that households know that their rate of time preference depends on the stock of the aggregate renewable resource. However, the households believe that they cannot control the motion of the aggregate renewable resource because they think that their impact is negligible. This argument follows those used by Lines (2005) and Yanase (2011).[7] Hence, when the households optimize the problem, they take it as given due to their negligibility.

The Hamiltonian function is $H_B \equiv u(C_t^y) + v(N_t) + w(C_t^n) + q_t \times (3) + \eta_t \times (1) + \mu_t \times (4)$, where q_t, η_t and μ_t represent the costate variables associated with each

[7] In papers based on consumption and capital externalities, a similar argument is used. That is, a household's utility is affected by the average levels of consumption or capital as well as private consumption (e.g., Futagami and Shibata, 1998; Liu and Turnovsky, 2005). These authors assume that households believe that they cannot influence aggregate consumption or aggregate capital stock, so that the average levels of these variables are exogenously given when they make their optimization decisions.

dynamic equation. The first-order conditions are:

$$\frac{\partial H_B}{\partial C_t^y} = 0 : \quad u'(C_t^y) = q_t, \tag{7a}$$

$$\frac{\partial H_B}{\partial C_t^n} = 0 : \quad w'(C_t^n) = pq_t, \tag{7b}$$

$$\frac{\partial H_B}{\partial I_t^y} = 0 : \quad q_t = \mu_t, \tag{7c}$$

$$\frac{\partial H_B}{\partial Z_t} = 0 : \quad q_t p h'(Z_t) = \eta_t, \tag{7d}$$

$$\frac{\partial H_B}{\partial B_t} = 0 : \quad r = -\frac{\dot{q}_t}{q_t} + \dot{\Theta}_t, \tag{7e}$$

$$\frac{\partial H_B}{\partial K_t} = 0 : \quad \frac{q_t R_t}{\mu_t} - \delta = -\frac{\dot{\mu}_t}{\mu_t} + \dot{\Theta}_t, \tag{7f}$$

$$\frac{\partial H_B}{\partial N_t} = 0 : \quad \frac{v'(N_t)}{\eta_t} + G'(N_t) = -\frac{\dot{\eta}_t}{\eta_t} + \dot{\Theta}_t. \tag{7g}$$

The transversality conditions are:

$$\lim_{t \to \infty} q_t B_t e^{-\Theta_t} = 0, \ \lim_{t \to \infty} \eta_t N_t e^{-\Theta_t} = 0, \ \text{and} \ \lim_{t \to \infty} \mu_t K_t e^{-\Theta_t} = 0. \tag{7h}$$

2.2 Equilibrium

In this subsection, we show that there exists a unique steady state that is characterized by indeterminacy. Assuming for simplicity that the total number of households is unity, it follows that because the level of the natural resource held by the representative household is equal to the economy's total stock of the natural resource, then $N_t = N_{\text{all},t}$ along the equilibrium path. Hence, the endogenous rate of time preference depends on N_t.

From (7c) we can show that $\frac{\dot{\mu}_t}{\mu_t} = \frac{\dot{q}_t}{q_t}$. Hence, from (7$e$) and (7$f$), $r = R_t - \delta$. Therefore, the competitive rental rate of capital is fixed at $R = R_t$. Consequently, the ratio of capital to labor $k = k_t = \frac{K_t}{l_t}$ is fixed, and the wage rate is also fixed $W = W_t$ (from (2)).

From (7a) and (7e), we derive the following Euler equation for consumption C_t^y:

$$\frac{\dot{C}_t^y}{C_t^y} = -\frac{u'(C_t^y)}{C_t^y u''(C_t^y)}(r - \rho(N_t)). \tag{8}$$

Using $(7a)$ and $(7d)$ yields:

$$\eta_t = ph'(Z_t)u'(C_t^y). \tag{9a}$$

Therefore, we can show that:

$$\frac{\dot{\eta}_t}{\eta_t} = \frac{h''(Z_t)\dot{Z}_t}{h'(Z_t)} + \frac{u''(C_t^y)\dot{C}_t^y}{u'(C_t^y)}. \tag{9b}$$

By substituting $(7a)$, $(9a)$ and $(9b)$ into $(7g)$, we can show that the dynamic motion of the harvested resource is as follows:

$$\frac{\dot{Z}_t}{Z_t} = -\frac{h'(Z_t)}{Z_t h''(Z_t)} \left(G'(N_t) + \frac{v'(N_t)}{ph'(Z_t)u'(C_t^y)} - r \right). \tag{10}$$

Thus, in our model, the economy's behavior is represented entirely by equations (1), (8) and (10).

Let us denote steady-state levels by using an asterisk. Then, a stationary solution $(C_t^{y,*}, N^*, Z^*)$ is characterized by $\dot{C}_t^y = \dot{N}_t = \dot{Z}_t = 0$ as follows:

$$r = \rho(N^*), \tag{11a}$$

$$G(N^*) = Z^*, \tag{11b}$$

$$r - G'(N^*) = \frac{v'(N^*)}{ph'(Z^*)u'(C^{y,*})}. \tag{11c}$$

The following proposition shows the uniqueness of the steady state in this economy.

Proposition 1. *Assume that $r > \rho(0)$ and $r > G'(\rho^{-1}(r))$. Then, there exists a unique steady state in this economy.*

Proof. Noting that $r > \rho(0)$, from $(11a)$, the steady-state level of the natural resource is uniquely determined. From $(11b)$, the unique level of the harvested natural resource in the steady state is determined. Then, assuming that $r > G'(\rho^{-1}(r))$, the left-hand side of $(11c)$ is positive. This means that the steady-state level of private consumption $C^{y,*}$ can be determined because the function $u(\cdot)$ satisfies the Inada conditions. It is then easy to show that $C^{y,*}, N^*$ and Z^* determine unique steady-state levels of $C^{n,*}$, $I^{y,*}$, K^* and B^*.[8]□

We examine the stability of the steady state. In the basic framework, let J_B denote the Jacobian matrix of the dynamic equations linearized around the steady state as follows:

$$\begin{bmatrix} \dot{C}^y_t \\ \dot{N}_t \\ \dot{Z}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})} & 0 \\ 0 & G'(N^*) & -1 \\ \frac{\partial \dot{Z}_t}{\partial C^y_t} & \frac{\partial \dot{Z}_t}{\partial N_t} & r - G'(N^*) \end{bmatrix}}_{J_B} \begin{bmatrix} C^y_t - C^{y,*} \\ N_t - N^* \\ Z_t - Z^* \end{bmatrix}, \tag{12}$$

where

$$\frac{\partial \dot{Z}_t}{\partial C^y_t} = \frac{h'(Z^*)u''(C^{y,*})}{h''(Z^*)u'(C^{y,*})}(r - G'(N^*))(> 0), \tag{13a}$$

$$\frac{\partial \dot{Z}_t}{\partial N_t} = -\frac{h'(Z^*)}{h''(Z^*)}\left(G''(N^*) + \frac{v''(N^*)(r - G'(N^*))}{v'(N^*)}\right)(< 0). \tag{13b}$$

Denoting by λ_B the eigenvalue of the system J_B, we obtain the following characteristic equation:

$$-\lambda_B^3 + r\lambda_B^2 - F_B\lambda_B^1 + \mathrm{Det}J_B = 0, \tag{14}$$

where

$$\mathrm{Det}J_B = -\frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})} \times \frac{\partial \dot{Z}_t}{\partial C^y_t}(> 0) \tag{15a}$$

$$F_B = -\frac{h'(Z^*)G''(N^*)}{h''(Z^*)} + \frac{v'(N^*)}{ph'(Z^*)u'(C^{y,*})}\left(\underbrace{G'(N^*) - \frac{h'(Z^*)v''(N^*)}{h''(Z^*)v'(N^*)}}_{(\#1)}\right). \tag{15b}$$

Then, we can prove the following proposition.

Proposition 2. *If* (#1) *in* (15b) *is negative, the steady-state equilibrium is indeterminate.*

Proof. We apply 'Ruth's theorem' to a third-order polynomial to show that the

[8]Hereafter, we omit proofs of the determination of the steady-state levels of the other variables, except for those used in the system, because their determination follows from the relevant first-order conditions.

number of roots of the polynomial given by (14) with positive real parts is equal to the number of variations of sign in the scheme:[9]

$$-1 \quad r \quad -F_B \;\; \mathrm{Det} J_B.$$

Det J_B and Tr $J_B(=r)$ have positive signs. If the sign of (#1) is negative, the sign of F_B given by (15b) is also negative. As a result, there exist one positive real root and two complex roots with negative real parts. Because the economy has one stock variable, N_t, and two nonpredetermined variables, C_t^y and Z_t, the dynamic character of the steady state exhibits indeterminacy. □

Proposition 2 shows that the negative sign of (#1) is a sufficient condition for the emergence of indeterminacy, which can alternatively be expressed as follows:

$$(\#1) \leq 0 \quad \Rightarrow \quad \frac{G'(N^*)N^*}{G(N^*)} \leq \underbrace{\frac{h'(Z^*)}{Z^*h''(Z^*)} \times \frac{v''(N^*)N^*}{v'(N^*)}}_{(+)} \equiv \gamma, \tag{16}$$

for which we have used (11b).

The right-hand side of (16) is composed of the product of two elasticities; namely, the elasticities of marginal utility and the marginal product with respect to the harvested natural resource. These elasticities, denoted by γ, are constant under the standard functions (such as HARA preferences and CES-type production functions). Alternatively, the left-hand side of (16) is a kind of elasticity of regeneration. Given the assumptions on $G(n)$, the elasticity of regeneration would not be constant. When the specified functions are used, it can be shown that the emergence of indeterminacy depends on the constant elasticity parameters and the steady-state level of the natural resource. For instance, following Brander and Scott-Taylor (1997) and Elíasson and Turnovsky (2004), the reproduction function is given by $G(N_t) = a \times N_t \left(1 - \frac{N_t}{\bar{N}}\right)$, in which the parameter a is the intrinsic growth rate of the resource. Because $\frac{G'(N)N}{G(N)} = \frac{1-\frac{2N}{\bar{N}}}{1-\frac{N}{\bar{N}}}$, the inequality (16) can be rewritten as:

$$1 - \frac{2N^*}{\bar{N}} \leq \gamma \left(1 - \frac{N^*}{\bar{N}}\right). \tag{17}$$

[9] See Theorem 1 of Benhabib and Perli (1994, p. 139).

Because the sign of $G(N^*)$ is positive, given $(11b)$, it follows that $N^* < \bar{N}$, which implies that the right-hand side of (17) is positive. If $N^* \geq \frac{\bar{N}}{2}$, so that the left-hand side of (17) is negative, the inequality (17) is satisfied, in which case, the steady state is indeterminate. This means that a resource-rich economy, for which $N^* \geq \frac{\bar{N}}{2}$ is satisfied, experiences a high degree of volatility, and thus faces the indeterminacy problem. By contrast, when $N^* < \frac{\bar{N}}{2}$, the economy can be unstable because (17) may not hold, in which case, F_B in $(15b)$ could have a positive sign.

More concretely, noting that $G'(N^*) \leq (>)0$ when $N^* \geq (<)\frac{\bar{N}}{2}$ in (17), we can summarize the above analysis as follows. If $G'(N^*)$ is negative, the condition (16) is satisfied. In other words, when the natural resource is abundant, so that $G'(N^*)$ is negative, the path of convergence toward the steady-state equilibrium is indeterminate. Furthermore, even if the natural resource is not sufficiently abundant for $G'(N^*) > 0$, the low level of marginal reproduction would satisfy the inequality (16). Thus, a greater abundance of natural resource wealth makes the small open economy more volatile.

To clarify how indeterminacy arises, let us first consider a case in which $G'(N^*) < 0$. In this case, because the condition (16) is satisfied, the equilibrium path is indeterminate. Suppose that the economy is initially at the steady state. Assume furthermore that a sunspot shock hits the economy such that households expect world interest rates to rise. Such a rise in the interest rate generates the negative rate of growth on the harvest of the natural resource, according to (10), which implies that agents are more willing to harvest the resource today. This is because the increase in bond returns causes a relative decline in the value of accumulating the natural resource. Therefore, the rise in the world interest rate directly decreases the level of the natural resource. However, the decrease in the natural resource generates more regeneration in the range within which $\frac{\partial G(N_t)}{\partial N_t} < 0$. Hence, the equilibrium path toward the unique steady state is indeterminate. Second, consider the case in which $G'(N^*)$ takes a sufficiently small positive value to satisfy (16). Then, because the positive value of $G'(N^*)$ is small, regeneration of the natural resource would remain high. The previous explanation also applies to this case.

If the steady-state level of the natural resource is sufficiently low for the positive value of $G'(N^*)$ to be sufficiently high, the steady state becomes unstable because F_B in (15b) may be positive. That is, when the steady-state level of the natural resource is very low, the economy may no longer converge to the steady-state equilibrium.

2.3 Withdrawal from the global resource market

When both the natural and nonnatural resource sectors have access to the global market, volatility is enhanced and the path to convergence may be indeterminate. In particular, the greater is natural resource wealth, the more likely is indeterminacy. This means that a resource-wealthy country may experience the resource curse in the sense that the economy, whose equilibrium path depends on household beliefs, experiences high volatility. In this subsection, we investigate whether volatility is reduced, in which case the indeterminacy problem is resolved, if the resource sector withdraws from the global market (with domestic residents still having access to a risk-free bond).

We assume that the commodity produced by using the renewable resource cannot be stored. Thus, domestic residents only harvest the amount they want to consume. In that case, the following equation is satisfied in every period:[10]

$$h(Z_t) = C_t^n. \tag{18}$$

By substituting (18) into (3) and (5), from the maximization problem, we obtain:

$$\text{Max} \;\; U_{B,L} \equiv \int_0^{+\infty} [u(C_t^y) + v(N_t) + w(Z_t)] \exp[-\Theta_t] dt, \tag{19}$$

subject to (1), (4) and:

$$\dot{B}_t = rB_t + W_t + R_t K_t - C_t^y - I_t^y. \tag{20}$$

Note that from $w(Z_t) \equiv w(h(Z_t))$, we can show that $w'(Z_t) = w'(h)h'(Z_t) > 0$ and $w''(Z_t) = w''(h)h'(Z_t)^2 + w'(h)h''(Z_t) < 0$.

[10] In Section 3.2, this assumption is relaxed.

Then, we define the Hamiltonian function as $H_{B,L} \equiv u(C_t^y) + v(N_t) + w(Z_t) + q_t \times (20) + \eta_t \times (1) + \mu_t \times (4)$. In this case, the first-order conditions, $\frac{\partial H_{B,L}}{\partial C_t^y} = 0$, $\frac{\partial H_{B,L}}{\partial I_t^y} = 0$, $\frac{\partial H_{B,L}}{\partial B_t} = 0$, $\frac{\partial H_{B,L}}{\partial K_t} = 0$ and $\frac{\partial H_{B,L}}{\partial N_t} = 0$ are the same as $(7a)$, $(7c)$, $(7e)$, $(7f)$ and $(7g)$. However, $\frac{\partial H_{B,L}}{\partial Z_t}$, which shows that $w'(Z_t) = \eta_t$, is different from $(7d)$. Consequently, we need only rewrite the dynamic equation for harvesting of the natural resource, which is:

$$\frac{\dot{Z}_t}{Z_t} = -\frac{w'(Z_t)}{Z_t w''(Z_t)} \left(G'(N_t) - \rho(N_t) + \frac{v'(N_t)}{w'(Z_t)} \right). \tag{21}$$

Note that the dynamic equations for consumption C_t^y and the natural resource N_t are identical to (1) and (8).

Because the economy is fully represented by the pair of dynamic equations (1) and (21), the growth rate of consumption is not necessarily zero, even in the long run.[11] That is, positive consumption growth may occur endogenously if $r > \rho(N^*)$. When the solvency condition, $\lim_{t \to \infty} B_t e^{-rt} = 0$ is imposed, from (8) and (20) we can show that consumption initially jumps, as follows:

$$C_0^y = \frac{B_0 + \frac{W + RK - I^y}{r}}{\int_0^\infty \left(e^{-\int_0^s \frac{u'(C_v^y)(r - \rho(N_v))}{C_v^y u''(C_v^y)}} dv \right) e^{-rs} ds}, \tag{22}$$

where the levels of capital stock K and investment I^y are constant along the equilibrium path. Contrast this with the analysis of Section 2.2, in which, because we assume that the resource sector has access to the global market, the constant world interest rate determines the steady-state level of the natural resource from $\dot{C}_t^y = 0$; that is, the growth rate of consumption is necessarily zero in the long run.

We obtain the following conclusion.

Proposition 3. *If the condition* (16) *is satisfied, the long-run levels of the pair* (N^*, Z^*) *are determined and the equilibrium path satisfies saddle-point stability.*

Proof. The equations $\dot{Z}_t = 0$ and $\dot{N}_t = 0$ are given by:

$$\rho(N^*) - G'(N^*) - \frac{v'(N^*)}{w'(Z^*)} = 0, \tag{23a}$$

[11] Both the stock and harvesting of the natural resource cannot continue to increase because the former is finite; that is, in the long run, $\dot{Z}_t = 0$ and $\dot{N}_t = 0$.

$$Z^* = G(N^*). \tag{23b}$$

By substituting (23b) into (23a), we can show that:

$$\Psi_{B,L}(N^*) \equiv \rho(N^*) - G'(N^*) - \frac{v'(N^*)}{w'(G(N^*))} = 0, \tag{24a}$$

where $\Psi'_{B,L}(N^*)$ is given by:

$$\Psi'_{B,L}(N^*) = \frac{v'(N^*)w''(G(N^*))}{w'(G(N^*))^2}\left\{\frac{(\rho'(N^*) - G''(N^*))w'(G(N^*))^2}{v'(N^*)w''(G(N^*))} \underbrace{- \frac{v''(N^*)w'(G(N^*))}{v'(N^*)w''(G(N^*))} + G'(N^*)}_{(\#2)}\right\}. \tag{24b}$$

Note that (#2) is negative under condition (16).

Because $\lim_{N^*\to 0}\Psi_{B,L}(N^*) = \rho(0) - G'(0) - \infty < 0$ and $\lim_{N^*\to\infty}\Psi_{B,L}(N^*) = \rho(\infty) - G'(\infty) > 0$, where $G'(\infty) < 0$, condition (16), which shows that $\Psi'_{B,L}(N^*)$ is positive, guarantees the existence of the steady state.

By taking linear approximations of (1) and (21), we obtain the following:

$$\begin{bmatrix}\dot{Z}_t \\ \dot{N}_t\end{bmatrix} = \underbrace{\begin{bmatrix}\frac{v'(N^*)}{w'(Z^*)} & -\frac{w'(Z^*)}{w''(Z^*)}\left(G''(N^*) - \rho'(N^*) + \frac{v''(N^*)}{w'(Z^*)}\right) \\ -1 & G'(N^*)\end{bmatrix}}_{J_{B,L}} \begin{bmatrix}Z_t - Z^* \\ N_t - N^*\end{bmatrix}. \tag{25}$$

Then, we can show that Tr $J_{B,L} = \rho(N^*) > 0$. In addition, the determinant is given by:

$$\begin{aligned}\mathrm{Det}J_{B,L} &= \frac{G'(N^*)v'(N^*)}{w'(Z^*)} - \frac{w'(Z^*)}{w''(Z^*)}\left(G''(N^*) - \rho'(N^*) + \frac{v''(N^*)}{w'(Z^*)}\right), \\ &= \frac{v'(N^*)}{w'(Z^*)}\left(\underbrace{G'(N^*) - \frac{v''(N^*)w'(Z^*)}{v'(N^*)w''(Z^*)}}_{(\#3)}\right) - \frac{w'(Z^*)}{w''(Z^*)}\left(G''(N^*) - \rho'(N^*)\right)(< 0),\end{aligned} \tag{26}$$

where under condition (16), the sign of (#3) is negative.

Because both the number of stock and control variables is one, the negative sign of the determinant shows that the steady state satisfies saddle-path stability. □

Let us summarize our findings from this section. First, our main finding is that the greater is a country's natural resource wealth, the more likely is its equilibrium path to be indeterminate when the resource sector has access to the global market. If the resource sector withdraws from the global resource market, the economy has a stable saddle path. Second, equation (16) reveals that the larger is the elasticity of marginal utility with respect to the natural resource, the more volatile is the economy when the resource sector has access to the global market. If the resource sector withdraws from the global market, economic volatility is lower the higher is the marginal utility elasticity. Third, the smaller is the elasticity of marginal production with respect to the natural resource, the more likely is indeterminacy when the resource sector has access to the global resource market. The withdrawal of the resource sector from the global market leads to saddle-point stability.

3 Extensions

In this section, we consider two extensions. We incorporate into our basic framework endogenous labor supply and investment in the natural resource. We show that our findings are robust to these extensions.

3.1 Labor shifts between production sectors and labor–leisure choice

We investigate how the abundance of a natural resource affects the allocation of labor among sectors and the stability of the economy. Assuming that labor supply is endogenously given, we examine whether our findings still hold.

We suppose that capital K_t and labor l_t are used in the nonnatural resource sector. In the resource sector, we assume that the harvesting of the natural resource requires labor input L_t as follows:

$$Z_t = \phi(L_t), \quad \phi'(\cdot) > 0, \quad \phi''(\cdot) < 0, \quad \phi(0) = 0. \tag{27}$$

In terms of modeling the allocation of labor supply between the natural and nonnatural resource sectors in a small open economy, this setup closely resembles that used by Elíasson and Turnovsky (2004). More generally however, individuals also face a labor–leisure choice. In other words, leisure time is given by $1 - L_t - l_t$.

Participation in the global resource market: We allow for endogenous labor–leisure choice. Thus, the maximization problem of the household is:

$$\text{Max: } U_L \equiv \int_0^{+\infty} [u(C_t^y) + v(N_t) + w(C_t^n) + \chi(1 - l_t - L_t)] \exp[-\Theta_t] dt, \tag{28}$$

subject to

$$\dot{B}_t = rB_t + W_t l_t + R_t K_t + ph(\phi(L_t)) - C_t^y - pC_t^n - I_t^y, \tag{29a}$$

$$\dot{N}_t = G(N_t) - \phi(L_t), \tag{29b}$$

and subject to the capital accumulation equation (4), in which the utility function of leisure $\chi(\cdot)$ satisfies $\frac{\partial \chi(1-l_t-L_t)}{\partial(1-l_t-L_t)} > 0$ and $\frac{\partial^2 \chi(1-l_t-L_t)}{\partial(1-l_t-L_t)^2} < 0$ as well as the Inada conditions.

By defining the Hamiltonian function as $H_L \equiv u(C_t^y) + v(N_t) + w(C_t^n) + \chi(1 - l_t - L_t) + q_t \times (29a) + \eta_t \times (29b) + \mu_t \times (4)$, we obtain the following necessary conditions:

$$\frac{\partial H_L}{\partial l_t} = 0 : \quad q_t W_t = \chi'(1 - l_t - L_t), \tag{30a}$$

$$\frac{\partial H_L}{\partial L_t} = 0 : \quad q_t ph'(L_t) - \eta_t \phi'(L_t) = \chi'(1 - l_t - L_t). \tag{30b}$$

In addition, the remaining first-order conditions, $\frac{\partial H_L}{\partial C_t^y} = 0$, $\frac{\partial H_L}{\partial C_t^n} = 0$, $\frac{\partial H_L}{\partial I_t^y} = 0$, $\frac{\partial H_L}{\partial B_t} = 0$, $\frac{\partial H_L}{\partial K_t} = 0$ and $\frac{\partial H_L}{\partial N_t} = 0$ are the same as $(7a)$, $(7b)$, $(7c)$, $(7e)$, $(7f)$ and $(7g)$. Note that, as shown in Section 2.2, the first-order condition for I_t^y yields $\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\dot{q}_t}{q_t}$ so that $R_t - \delta = r$ through time, which leads to $R_t = R$, and thus $k_t = k$ and $W_t = W$.

Using $\frac{\partial H_L}{\partial C_t^y}$, $(30a)$ and $(30b)$ yields:

$$\eta_t = \frac{u'(c_t^y)\,(ph'(\phi)\phi'(L_t) - W)}{\phi'(L_t)}, \tag{31a}$$

where η_t is positive so that $ph'(\phi)\phi'(L_t) > W$ in equilibrium. Moreover, by differentiating $(31a)$ with respect to time, we obtain the following:

$$\dot{\eta}_t = \frac{(ph'(\phi)\phi'(L_t) - W)u'(C_t^y)(\rho(N_t) - r)}{\phi'(L_t)} - u'(C_t^y)\beta_L(L_t)\dot{L}_t, \tag{31b}$$

where $\beta_L(L_t)$ is given by:

$$\beta_L(L_t) \equiv -\frac{ph''(\phi)\phi'(L_t)^3 + W\phi''(L_t)}{ph'(\phi)\phi'(L_t) - W}(> 0).$$

Incorporating (31a) and (31b) into $\frac{\partial H_L}{\partial N_t}$, the dynamic behavior of labor supply is:

$$\dot{L}_t = \frac{1}{\beta_L(L_t)}\left\{G'(N_t) - r + \frac{\phi'(L_t)v'(N_t)}{u'(C_t^y)(ph'(\phi)\phi'(L_t) - W)}\right\}. \tag{32}$$

Macroeconomic equilibrium can be expressed by the differential equations for C_t^y, N_t and L_t (8), (29b) and (32).[12] This leads us to the following result.

Proposition 4. *Suppose that $r > \rho(0)$, $G(\bar{N}) < \phi(1)$ and $r > G'(\rho^{-1}(r))$. Then, there exists a unique steady-state equilibrium. The steady state is indeterminate if the following inequality is satisfied:*

$$G'(N^*) \leq -\frac{\phi'(L^*)v''(N^*)}{v'(N^*)\beta_L(L^*)}, \quad \Rightarrow \quad \frac{G'(N^*)N^*}{G(N^*)} \leq \underbrace{-\frac{\phi'(L^*)}{\phi(L^*)\beta_L(L^*)} \times \frac{v''(N^*)N^*}{v'(N^*)}}_{(+)}. \tag{33}$$

Proof. As in Proposition 1, $\dot{C}_t^y = 0$ determines the steady-state level of the natural resource under the assumption that $r > \rho(0)$. Then, with $\phi(0) = 0$ and $G(\bar{N}) < \phi(1)$, a monotonic increase in $\phi(\cdot)$ determines the steady-state level of labor supply L^* from $\dot{N}_t = 0$. Then, assuming that $r > G'(\rho^{-1}(r))$, the standard assumption in the context of the utility function, $u'(\cdot) < 0$ pins down a positive level of $C^{y,*}$.

Taking a linear approximation around the steady state leads to the following:

$$\begin{bmatrix} \dot{C}_t^y \\ \dot{N}_t \\ \dot{L}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})} & 0 \\ 0 & G'(N^*) & -\phi'(L^*) \\ \frac{\partial \dot{L}_t}{\partial C_t^y} & \frac{\partial \dot{L}_t}{\partial N_t} & r - G'(N^*) \end{bmatrix}}_{J_L} \begin{bmatrix} C_t^y - C^{*,y} \\ N_t - N^* \\ L_t - L^* \end{bmatrix}, \tag{34}$$

where

$$\frac{\partial \dot{L}_t}{\partial C_t^y} = \frac{(r - G'(N^*))u''(C^{y,*})}{(ph''(Z^*)\phi'(L^*)^3 + W\phi''(L^*))u'(C^{y,*})}(> 0),$$

[12] From $\frac{\partial H_L}{\partial C_t^y} = 0$ and $\frac{\partial H_L}{\partial B_t} = 0$, which are identical to (7a) and (7e), we obtain the Euler equation for consumption C_t^y: $\frac{\dot{C}_t^y}{C_t^y} = -\frac{u'(C_t^y)(r-\rho(N_t))}{C_t^y u''(C_t^y)}$.

$$\frac{\partial \dot{L}_t}{\partial N_t} = \frac{1}{\beta_L(L^*)}\left\{G''(N^*) + \frac{v''(N^*)(r - G'(N^*))}{v'(N^*)}\right\}(< 0).$$

The characteristic equation is given by $|J_L - \lambda_L I| = -\lambda_L^3 + r\lambda_L^2 - F_L\lambda_L^1 + \mathrm{Det} J_L = 0$, where

$$\mathrm{Det}\ J_L = \frac{\partial \dot{L}_t}{\partial C_t^y} \times \frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})} \times (-\phi'(L^*))(> 0), \tag{35a}$$

$$F_L = \frac{\phi'(L^*)G''(N^*)}{\beta_L(L^*)} + (r - G'(N^*))\Bigg(\underbrace{G'(N^*) + \frac{\phi'(L^*)v''(N^*)}{v'(N^*)\beta_L(L^*)}}_{(\#4)}\Bigg)(< 0), \tag{35b}$$

where (#4) < 0 from (33).

Given that the trace and determinant are positive and F_L in (35*b*) is negative, the path of convergence toward the steady state is indeterminate. □

Withdrawal from the global resource market: If we assume that the economy does not export its natural resource, then $h(\phi(L_t)) = C_t^n$ as in (18). By maximizing (28) subject to (4), (29*a*) and (29*b*) under $h(\phi(L_t)) = C_t^n$, we obtain the first-order conditions. The first-order condition with respect to L_t, which is given by $w'(h)h'(\phi)\phi'(L_t) - \eta_t\phi'(L_t) = \chi'(1 - l_t - L_t)$, is different from (30*b*). Thus, in this setup, η_t and $\dot{\eta}_t$ are:

$$\eta_t = w'(h)h'(\phi) - \frac{u'(C_t^y)W}{\phi'(L_t)}, \quad \dot{\eta}_t = -\beta_{L,L}(C_t^y, L_t)\dot{L}_t - \frac{u'(C_t^y)(\rho(N_t) - r)}{\phi'(L_t)},$$

where $\beta_{L,L}(C_t^y, L_t)$ is:

$$\beta_{L,L}(C_t^y, L_t) = -w''(h)h'(\phi)^2\phi'(L_t) - w'(h)h''(\phi)\phi'(L_t) - \frac{u'(C_t^u)\phi''(L_t)W}{\phi'(L_t)^2}(> 0).$$

The dynamic equation for labor supply can be rewritten as:

$$\dot{L}_t = \frac{1}{\beta_{L,L}(C_t^y, L_t)}\left\{v'(N_t) - (\rho(N_t) - G'(N_t))w'(h)h'(\phi) + \frac{Wu'(C_t^y)(r - G'(N_t))}{\phi'(L_t)}\right\}. \tag{36}$$

The dynamic equations describing the economy are given by (8), (29*b*) and (36). Hence, we obtain the following result.

Proposition 5. *Suppose that* $r > \rho(0)$, $G(\bar{N}) < \phi(1)$ *and* $r > G'(\rho^{-1}(r))$. *Then, there exists a unique steady-state equilibrium that satisfies saddle-point stability.*

Proof. This is omitted because it is similar to the proof of Proposition 1.

Taking a linear approximation of this system yields:

$$\begin{bmatrix} \dot{C}_t^y \\ \dot{N}_t \\ \dot{L}_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})} & 0 \\ 0 & G'(N^*) & -\phi'(L^*) \\ \frac{\partial \dot{L}_t}{\partial C_t^y} & \frac{\partial \dot{L}_t}{\partial N_t} & r - G'(N^*) \end{bmatrix}}_{J_{L,L}} \begin{bmatrix} C_t^y - C^{y,*} \\ N_t - N^* \\ L_t - L^*, \end{bmatrix}$$

where

$$\frac{\partial \dot{L}_t}{\partial C_t^y} = \frac{W u''(C^{y,*})(r - G'(N^*))}{\beta_{L,L}(L^*, C^{y,*})\phi'(L^*)}(< 0),$$

$$\frac{\partial \dot{L}_t}{\partial N_t} = \frac{1}{\beta_{L,L}(L^*, C^{y,*})}\left\{ v''(N^*) + G''(N^*)\left(w'(h)h'(\phi) - \frac{W u'(C^{y,*})}{\phi'(L^*)} \right) - \rho'(N^*)w'(h)h'(\phi) \right\}(< 0).$$

Hence, the trace and the determinant are:

$$\text{Tr } J_{L,L} = G'(N^*) + \frac{\partial \dot{L}_t}{\partial L_t} = r(> 0), \quad \text{Det } J_{L,L} = \frac{\partial \dot{L}_t}{\partial C_t^y}\frac{u'(C^{y,*})\rho'(N^*)}{u''(C^{y,*})}(-\phi'(L^*))(< 0).$$

Given that there is one stock variable and two jump variables, Tr $J_{L,L} > 0$ and Det $J_{L,L} < 0$ ensure saddle-path stability. □

3.2 Investment in the natural resource

We now assume that firms in the resource sector produce not only the consumption good C_t^n but also the investment good I_t^n. Then, we can extend our basic framework to a model with two consumable goods and two investment goods. Specifically, incorporating investment in the natural resource means that households sow seeds or plant seedlings, which implies that households are motivated to nurture the resource. This extension yields a more general analysis of the dynamics of the natural resource.

Participation in the global resource market: The budget constraint and the evolution of the natural resource can be rewritten as:

$$\dot{B}_t = rB_t + W_t + R_t K_t + ph(Z_t) - C_t^y - I_t^y - pC_t^n - pI_t^n, \tag{37a}$$

$$\dot{N}_t = G(N_t) - Z_t + \Gamma(I_t^n), \tag{37b}$$

along with the capital accumulation equation (4).

Using the household utility function given by (5), the Hamiltonian function is defined as $H_I \equiv u(C_t^y)+v(N_t)+w(C_t^n)+q_t\times(37a)+\eta_t\times(37b)+\mu_t\times(4)$. In comparison with the basic framework, the first-order condition with respect to investment in the natural resource I_t^n is now given by:

$$\frac{\partial H_I}{\partial I_t^n} = 0: \quad \eta_t \Gamma'(I_t^n) = p q_t, \tag{38}$$

where $\frac{\partial H_I}{\partial C_t^y} = 0$, $\frac{\partial H_I}{\partial C_t^n} = 0$, $\frac{\partial H_I}{\partial Z_t} = 0$, $\frac{\partial H_I}{\partial B_t} = 0$, $\frac{\partial H_I}{\partial K_t} = 0$ and $\frac{\partial H_I}{\partial N_t} = 0$ are the same as $(7a)$, $(7b)$, $(7d)$, $(7e)$, $(7f)$ and $(7g)$, respectively. As a result, the dynamic equations for C_t^y and Z_t correspond to (8) and (10), respectively.

From (38) and $\frac{\partial H_I}{\partial Z_t}$, we can obtain the following:

$$h'(Z_t) = \frac{1}{\Gamma'(I_t^n)}. \tag{39a}$$

This leads to:

$$I_t^n = I_t^n(Z_t), \text{ where } \frac{\partial I_t^n}{\partial Z_t} = -\frac{\Gamma'(I_t^n)h''(Z_t)}{\Gamma''(I_t^n)h'(Z_t)}(<0), \ \lim_{Z_t\to 0} I_t^n(Z_t) = \infty, \text{ and } \lim_{Z_t\to\infty} I_t^n(Z_t) = 0. \tag{39b}$$

By incorporating $(39b)$ into (1), we can show that:

$$\dot{N}_t = G(N_t) - Z_t + \Gamma(I_t^n(Z_t)). \tag{40}$$

From (8), (10) and (40), we obtain the following proposition.

Proposition 6. *Suppose that $r > \rho(0)$ and $r > G'(\rho^{-1}(r))$. Then, there exists a unique steady-state equilibrium. The steady state is indeterminate if the following inequality is satisfied:*

$$\frac{G'(N^*)N^*}{G(N^*)+\Gamma(I^n(Z^*))} \le \underbrace{\frac{h'(Z^*)}{h''(Z^*)Z^*} \times \frac{v''(N^*)N^*}{v'(N^*)} \times \left(1 - \Gamma'(I^n)\frac{\partial I^{n,*}}{\partial Z^*}\right)}_{(+)}. \tag{41}$$

Proof. Because the only difference from the basic framework is the dynamic equation for the natural resource, we skip the proof because the procedures are similar to those used to prove Propositions 1 and 2. □

Withdrawal from the global resource market: We assume that the resource sector does not access the global market:

$$h(Z_t) = C_t^n + I_t^n, \ \Rightarrow \ C_t^n = h(Z_t) - I_t^n. \tag{42}$$

Unlike the models in Sections 2.3 and 3.1, in this subsection, part of the resource commodity is used to undertake investment in the natural resource.

Substituting equation (42) into the utility function yields:

$$U_{I,L} \equiv \int_0^{\infty} (u(C_t^y) + v(N_t) + w(h(Z_t) - I_t^n)) \exp[-\Theta_t] dt. \tag{43}$$

In addition, given (42), the budget constraint is identical to (20). As a result, the household maximizes (43) subject to (4), (20) and (37b), where $H_{I,L} \equiv u(C_t^y) + v(N_t) + w(h(Z_t) - I_t^n) + q_t \times (20) + \eta_t \times (37b) + \mu_t \times (4)$:

$$\frac{\partial H_{I,L}}{\partial Z_t}: \quad w'(h(Z_t) - I_t^n) h'(Z_t) = \eta_t, \tag{44a}$$

$$\frac{\partial H_{I,L}}{\partial I_t^n}: \quad w'(h(Z_t) - I_t^n) = \eta_t \Gamma'(I_t^n), \tag{44b}$$

where $\frac{\partial H_{I,L}}{\partial C_t^y} = 0$, $\frac{\partial H_{I,L}}{\partial I_t^y} = 0$, $\frac{\partial H_{I,L}}{\partial B_t} = 0$, $\frac{\partial H_{I,L}}{\partial K_t} = 0$ and $\frac{\partial H_{I,L}}{\partial N_t} = 0$ are identical to (7a), (7c), (7e), (7f) and (7g).

From (44a) and (44b), we obtain an equation that is identical to (39b). Hence, we can show that:

$$\eta_t = h'(Z_t) w'(h(Z_t) - I_t^n(Z_t)), \quad \dot{\eta}_t = -\beta_{I,L}(Z_t) \dot{Z}_t, \tag{45}$$

where

$$\beta_{I,L}(Z_t) = -\frac{h''(Z_t)}{h'(Z_t)} - \frac{w''(h(Z_t) - I_t^n(Z_t)) \left(h'(Z_t) - \frac{\partial I_t^n}{\partial Z_t} \right)}{w'(h(Z_t) - I_t^n(Z_t))} (> 0).$$

Consequently, we can derive the following dynamic equation for the harvested resource:

$$\dot{Z}_t = \frac{1}{\beta_{I,L}(Z_t)} \left(G'(N_t) - \rho(N_t) + \frac{v'(N_t)}{w'(h(Z_t) - I_t^n(Z_t)) h'(Z_t)} \right). \tag{46}$$

Moreover, the dynamic equation for the natural resource is the same as (40).

The steady-state equilibrium is given by $\dot{Z}_t = \dot{N}_t = 0$:

$$\rho(N^*) - G'(N^*) - \frac{v'(N^*)}{w'(h(Z^*) - I^n(Z^*)) h'(Z^*)} = 0, \tag{47a}$$

$$Z^* - \Gamma(I^n(Z^*)) = G(N^*). \tag{47b}$$

We can then state the following proposition in relation to the existence and stability of the steady state equilibrium.

Proposition 7. *When the following inequality is satisfied, there exist steady-state levels of* (Z^*, N^*) *and the equilibrium path exhibits saddle-path stability.*

$$\frac{G'(N^*)N^*}{G(N^*)} \leq \underbrace{\frac{v''(N^*)N^*}{v'(N^*)} \times \frac{-1+\Gamma'(I^{n,*})\frac{\partial I^{n,*}}{\partial Z^*}}{\beta_{I,L}(Z^*)(Z^*-\Gamma(I_t^n(Z^*)))}}_{(+)} \tag{48}$$

Proof. From (47b), we can show that $Z^* = Z(N^*)$, where $Z'(N^*) = \frac{G'(N^*)}{1-\Gamma'(I^{n,*})\frac{\partial I^n(Z^*)}{\partial Z^*}}$.[13] Substituting this equation into (47a) yields:

$$\Psi_{I,L}(N^*) \equiv \rho(N^*) - G'(N^*) - \frac{v^*(N^*)}{h'(Z(N^*))w'(h(Z(N^*)) - I^n(Z(N^*)))}. \tag{49}$$

As for Proposition 3, $\lim_{N^*\to 0}\Psi_{I,L}(N^*) = \rho(0) - G'(0) - \infty (< 0)$ and $\lim_{N^*\to\infty}\Psi_{I,L}(N^*) = \rho(\infty) - G'(\infty)(> 0)$, where $G'(\infty) < 0$.[14]

$\Psi'_{I,L}(N^*)$ is given by:

$$\Psi'_{I,L}(N^*) = \rho' - G'' - \frac{v'}{w'h'}\Bigg\{\underbrace{\frac{v''}{v'} - \frac{G'}{1-\Gamma'\frac{\partial I^n(Z^*)}{\partial Z^*}}\left(\frac{w''\left(h' - \frac{\partial I^n(Z^*)}{\partial Z^*}\right)}{w'} + \frac{h''}{h'}\right)}_{(\#5)}\Bigg\}. \tag{50}$$

For $\Psi'_{I,L}(N^*)$ to be positive, (#5) must be negative. Hence, by rearranging (#3) and making use of (48), we can prove the existence of the steady state.

To confirm the stability of the unique steady state, we use the linear approximation for (40) and (46):

$$\begin{bmatrix}\dot{Z}_t \\ \dot{N}_t\end{bmatrix} = \underbrace{\begin{bmatrix}\rho - G' & \frac{1}{\beta_{I,L}}\left(G'' - \rho' + \frac{v''}{w'h'}\right) \\ -1+\Gamma'\frac{\partial I^n(Z^*)}{\partial Z^*} & G'\end{bmatrix}}_{J_{I,L}}\begin{bmatrix}Z_t - Z^* \\ N_t - N^*\end{bmatrix}. \tag{51}$$

[13]When $N^* \to \infty$ or $N^* \to 0$ so that $G = 0$, it follows that $Z^* = \Gamma(I^n(Z^*))$. In this time period, Z^* converges to a finite level.

[14]As in the basic framework, $r > \rho(N^*)$ implies a positive consumption growth rate, C_t^y.

It transpires that Tr $J_{I,L} = r(>0)$. Furthermore, by using (47a), we obtain:

$$\text{Det } J_{I,L} = \underbrace{-\frac{\left(-1+\Gamma'\frac{\partial I^n}{\partial Z}\right)(G''-\rho')}{\beta_{I,L}}}_{(-)} + (\rho - G')\left\{\underbrace{G' - \frac{\left(-1+\Gamma'\frac{\partial I^n}{\partial Z}\right)v''}{v'\beta_{I,L}}}_{(\#6)}\right\}(<0),$$

where (48) implies that (#6) is negative. Hence, the determinant is negative. Consequently, this ensures saddle-path stability. □

The findings of this section can be interpreted in the same manner as those based on the basic framework. Assume that the resource sector has access to the global market. Then, even when endogenous labor supply or investment in the natural resource is incorporated, we conclude that a country that is sufficiently rich in natural resources for G' to be small or negative exhibits high volatility and hence an indeterminate equilibrium. Alternatively, when the economy has no access to the global resource market, the steady state is determinate. Furthermore, conditions (16), (33), (41) and (48) still imply that a large elasticity of marginal utility with respect to the natural resource, $\frac{v''(N^*)N^*}{v'(N^*)}$, generates high volatility when the resource sector is open, but low volatility when it is closed to global market. While both endogenous labor supply and investment in the natural resource contribute to the emergence of indeterminacy, both make the resource sector more complicated, by incorporating problems such as time allocation and investment in the resource. As a result, conditions (33), (41) and (48) would be more complicated.

4 Conclusion

In this paper, we analyzed how the presence of the global resource market affects macroeconomic stability in a dynamic model that incorporates a renewable resource sector. Our findings reveal that when the resource sector has access to the global market, the economy faces a high degree of volatility, which causes the equilibrium path toward the steady state to become indeterminate or unstable. If, instead, the resource sector withdraws from the global market, volatility is reduced and the economy finds saddle-path stability. This finding does not change even if labor shifts between production sectors and labor–leisure choice are incorporated, or if investment in the natural resource is added.

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