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Favoritism toward the Poor and a Discontinuous Tax Structure

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Abstract

The aim of this paper is to theoretically clarify the following two points. First, even though the government shows favoritism to the poor and wants to exempt low-income taxpayers and secure the necessary income tax revenue by taxing only high-income taxpayers, it ends up taxing the poor, which is in opposition to favoritism, because of its inability to observe individual taxpayer's income level. Second, even without observing each taxpayer's income level, if favoritism is sufficiently strong, the government can discontinuously resolve such unintentional taxation.

Keywords: Favoritism, Optimal income taxation, Tax evasion, Nash equilibrium.

JEL Classification: H21, H26, D82

1. Introduction

The aim of this paper is to clarify two points from the standpoint of microeconomic theory. First, even though the government makes the value judgment of giving preference (i.e., favoritism) to the poor and wants to exempt low-income earners and secure the necessary income tax revenue by taxing only high-income earners, it ends up taxing the poor, which is in direct opposition to favoritism, because it is unable to directly observe each taxpayer's income level. Second, even without observing each taxpayer's income level, if favoritism is sufficiently strong, the government can discontinuously resolve such unintentional taxation.

This paper, in which favoritism is represented by the weight that the government places on the income levels of the poor, shows that when this weight exceeds a certain threshold, a discontinuity exists in the tax structure that reduces the tax burden of the poor.

In addition, the model used in this study is premised on the strategic interaction between taxpayers who are concealing income and the tax authorities who are uncovering unreported income. Previous work on tax law enforcement based on this premise includes Greatz, Reinganum, and Wilde (1986) and a follow-up work by Chander and Wilde (1992). Both of these works exogenously provide an income tax structure (i.e., how much tax to impose on the different income levels of taxpayers) using comparative statistics in relation to the Nash equilibrium. In contrast, this paper presents a model, within the income tax revenue constraints of the Nash equilibrium, in which the government endogenously determines an income tax structure that focuses on the income levels of the poor rather than those of the affluent.

In general, the ability to enforce capital punishment is a crime deterrent, even in the absence of observability (Ehrlich, 1975). However, in actuality, such punishment is not imposed in tax evasion cases. The reasons for this have been discussed by Pestieau et al. (1994) among other works. The present paper focuses on how heavy maximum penalties on concealed income levels, rather than preventing tax evasion, affect the nature of tax revenue constraints.

This paper is organized as follows. The next section describes the game between taxpayers and tax authority, as well as the Nash equilibrium of the game. The third section presents the government’s decision tree in formulating a tax structure and shows a tax structure determined by government favoritism. Finally, the fourth section interprets the results and compares them with the case of perfect information.

2. The Game Between Taxpayers and Tax Authorities and the Nash Equilibrium

Let us consider a simultaneous-move game between multiple taxpayers and tax authority. The taxpayer population is normalized to 1. Among taxpayers, the proportion $q \in (0,1)$ has high incomes (I^H), while the remaining proportion $1-q$ has low incomes (I^L). Each taxpayer files a tax return. In other words, taxpayers with I^H (I^L) report their income to the authority and pay taxes of T^H (T^L) on their reported income.

Suppose income taxes are progressive (i.e., $T^H > T^L$). While the tax authority is aware of the abovementioned income distribution, we postulate that it is common knowledge that the tax authority does not know the income level of individual taxpayer. If high-income taxpayers pay cost K to conceal income and falsely declare a low income to the tax authority, they will be able to avoid paying taxes of $T = T^H - T^L$. Let the probability that a high-income taxpayer makes a false tax declaration be $\alpha \in [0,1]$.

Assigning a value of α arbitrarily using Bayes' theorem, the tax authority can estimate the proportion μ of tax evaders among high-income taxpayers as

$$\mu = q\alpha / \left(q\alpha + 1 - q\right). \tag{1}$$

Simultaneously, the tax authority can verify whether income has been concealed by spending cost C to conduct a tax audit. Let the probability of the tax authority conducting an audit be $\beta \in [0,1]$. The authority imposes $F < I = I^H - I^L$ on those found to be tax evaders. In addition, the tax authorities are later paid a reward by the government that is equivalent to the concealed uncovered income.

Figure 1 illustrates the model described above in the form of a game tree. The circled letters N, T, and A represent the moves of Nature, the taxpayer, and the tax Authority, respectively. The dotted line connecting two of the moves represents the information set. Further, *a* and *n* represent an audit or the lack of an audit, respectively. The expressions in parentheses represent, to the left of the comma, payoff to the taxpayer, and, to the right of the comma, payoff to the tax authority.

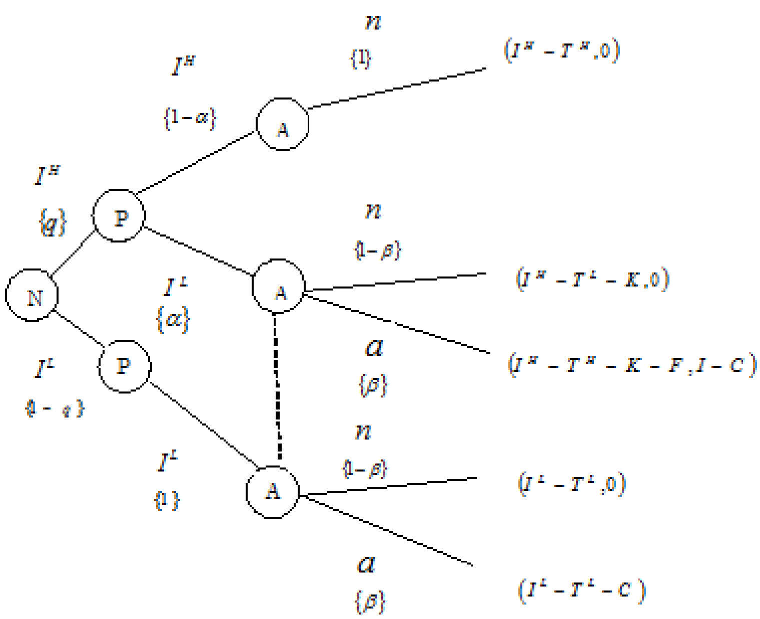

Figure 1. Game Tree

For simplification, both the taxpayers and the tax authority are assumed to be risk-neutral. In setting up this model, we can formulate the expected benefit for the high-income earner (U^H) as

$$\begin{aligned} U^H(\alpha,\beta) &= \alpha\left(\beta\left(I^H - T^H - F - K\right) + (1-\beta)\left(I^H - T^L - K\right)\right). \\ &+ (1-\alpha)\left(I^H - T^H\right) \qquad \forall \beta \in [\mathbf{0},\mathbf{1}] \end{aligned} \tag{2}$$

and the expected benefit for the tax authority (π) as

$$\pi(\alpha,\beta)=\beta(\mu(\alpha)I-C) \quad \forall\alpha\in[0,1]. \tag{3}$$

The partial derivatives for α in Equation (2) are linear in relation to β. Further, because $\frac{\partial U(\alpha,1)}{\partial\alpha}=-(F+K)$, if $T>K$, then $\beta^0\in(0,1)$ exists according to the intermediate value theorem. However, because $\beta^0=\frac{T-K}{T+F}$, the taxpayer's best reaction is

$$\alpha(\beta)\begin{cases} =1 & if \quad \beta<\beta^0 \\ \in[0,1] & if \quad \beta=\beta^0 \\ =0 & if \quad \beta>\beta^0 \end{cases}. \tag{4}$$

On the other hand, the tax authority's best reaction is

$$\beta(\alpha)\begin{cases} =1 & if \quad \mu(\alpha)>\mu^0 \\ \in[0,1] & if \quad \mu(\alpha)=\mu^0 \\ =0 & if \quad \mu(\alpha)<\mu^0 \end{cases}. \tag{5}$$

However, $\mu^0=C/I$. Because $\mu(\alpha)$ has been already defined, we can obtain

$\alpha^0 = \dfrac{(1-q)C}{q(I-C)}$. When $I > C/q$, $\alpha^o \in (0,1)$ exists. Therefore, the authority's best reaction is as follows: when $C < I$,

$$\beta(\alpha)\begin{cases} =1 & if \quad \alpha > \alpha^0 \\ \in [0,1] & if \; \alpha = \alpha^0 \\ =0 & if \quad \alpha < \alpha^0 \end{cases} \quad . \tag{6}$$

Equations (4) and (6) yield the following proposition.

Proposition 1: If $T > K$ and $I > C/q$, then the Nash equilibrium is a mixed strategy, i.e., (α^0, β^0).

3. Optimal Taxation Structure

In the game between taxpayers and the tax authority, if the Nash equilibrium is realized as (α^0, β^0), then as long as T^L, T^H, and F are given, the government is confronted with the following net expected tax revenue R^e:

$$R^e = q\alpha^0\beta^0\left(T^H + F\right) + \left(1-q\right)\beta^0 T^L$$
$$+\left(1-\beta^0\right)\left(q\alpha^0 + 1 - q\right)T^L + q\left(1-\alpha^0\right)T^H - q\alpha^0\beta^0 I \qquad (7)$$

The sum of the first four terms in the right-hand side is gross tax revenue, and the fifth term is the reward to the authority. In this paper, we suppose that the government's required tax revenues are substantial, and focus on a case in which the required tax revenues are financed solely by income tax revenues. Representing the required tax revenue as $\bar{R}^e$, we can obtain the following lemma for the constraints facing the government in determining the tax structure:

Lemma: When $R = \bar{R}^e$ and F is bounded, the set $\left(T, T^L\right)$ for the government's tax structure is strictly convex for T.

(For proof, see Appendix.)

To analyze the optimal tax structure under the abovementioned constraints, we define economic welfare W^e as the weighted sum of the payoff for truthful tax filers as

$$W^{e}=(1-\alpha)q\left(I^{H}-T^{H}\right)+\zeta(1-q)\left(I^{L}-T^{L}\right). \tag{8}$$

In this formalization, $\zeta>1$ represents government favoritism. That is, with regard to the benefit extended to accurate tax filers, when determining the tax structure, the government puts more pressure on low-income earners than on high-income earners.

Fixing W^{e} in the above equation as $\overline{W}>0$ and solving for T^{L} gives the following equation:

$$T^{L}=\frac{(1-\alpha)I^{H}+s(1-q)I^{L}-\overline{W}^{e}-(1-\alpha)qT}{(1-\alpha)q+\zeta(1-q)}. \tag{9}$$

The graph of this equation, which is in Quadrant I of the coordinates $\left(T,T^{L}\right)$, shifts to the southwest as $\overline{W}$ increases.

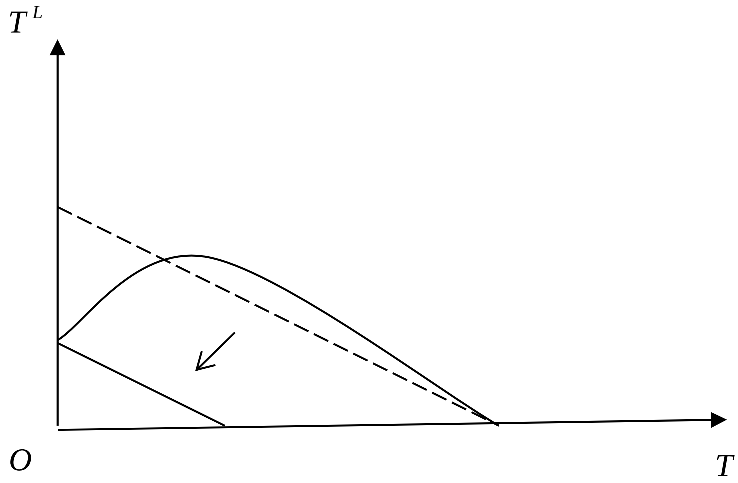

Figure 2-a. Weak Favoritism

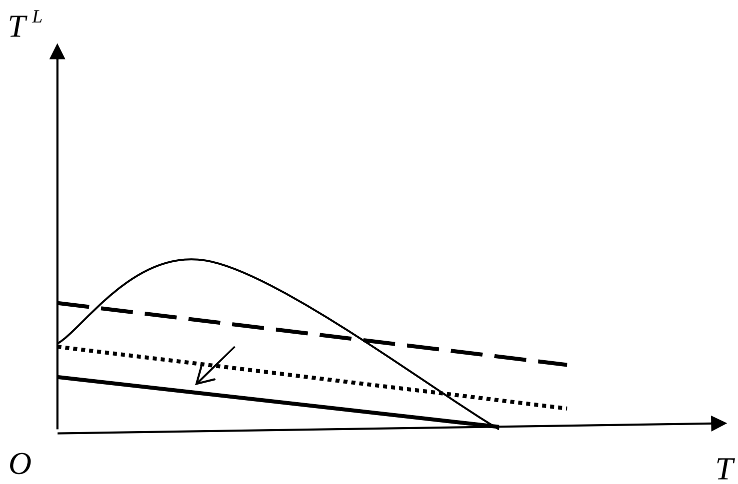

Figure 2-b. Strong Favoritism

When ζ is sufficiently small, Equation (9) is described as the solid line in Figure 2-a, and $T^H = T^L > 0$ at equilibrium. On the other hand, as ζ increases, the graph rotates counterclockwise and shifts to the solid line shown in Figure 2-b such that the optimal tax structure jumps to $T^H > T^L = 0$.

Thus, we formulate the following proposition using the Lemma and Equation (9).

Proposition 2: The tax structure that maximizes $\overline{W}^e$ under tax revenue constraints is one of the two corner solutions with regard to government's favoritism ζ.

Therefore, even if the government shows favoritism to the poor, when this favoritism is sufficiently weak, the same income tax is imposed on the poor as that on the affluent; however, as favoritism becomes stronger, the government can give tax exemptions to the poor even if information asymmetry exists.

4. Conclusion

When the government is unable to determine who is poor (i.e., when there is information asymmetry regarding income), the government may impose a very heavy income tax burden on low-income earners. In this paper, however, we have shown that this situation can be resolved through strong favoritism by the government.

With perfect information, the government can maximize $\bar{W} = q\left(I^H - T^H\right) + \zeta(1-q)\left(I^L - T^L\right)$ subject to tax revenue constraint $\bar{R} = qT^H + (1-q)T^L$ such that at equilibrium, $T^H > T^L = 0$ on the plane $\left(T^L, T^H\right)$. In other words, tax exemptions for the poor are realized in relation to favoritism ζ. Therefore, the results we obtained under the conditions of asymmetrical information show that the tax structure problem can be completely resolved, even if the government does not know which taxpayers are truly poor.

Appendix: Proof of Lemma

Setting $R^e = \overline{R} > 0$ and substituting $T^H = T + T^L$ and $\beta^0 = \dfrac{T-K}{T+F}$ in Equation (7), we obtain

$$T^L = \frac{-qT^2 - BT + D}{T+F} \qquad (*)$$

,where $B = qF - q(I-K)\alpha^0 - \overline{R}$, $D = -q(I-F)K\alpha^0 - F\overline{R}$. When $I > F$, then because $T^L(0) > 0$, there is an intercept on the vertical axis, as shown in the graph in Figure 2.

On differentiating the above equation, we get

$$dT^L / dT = \frac{-qT^2 + (B - 2qF)T + D}{(T+F)^2}.$$

To check the sign, we put the numerator on the right-hand side of G. When $\overline{R}$ is sufficiently large, $G(0) < 0$ and $dG / dT = -2q(T + F)T + B < 0$. Furthermore,

because T^0, which is satisfied with $dG / dT^0 = 0$, is negative, $G < 0$ for arbitrary $T > 0$.

Therefore, $dT^L / dT < 0$.

In solving for T^* which is satisfied with $dT^L / dT^* = 0$, we

get $T^* = -F + \dfrac{-B \pm \sqrt{(2qF - B)^2 + 4qD}}{-2q}$, so when $\bar{R}$ is large enough, $T^* > 0$. Therefore,

Equation (*) is strictly convex in relation to T.

Q.E.D.

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