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Optimal income tax structure with favoritism

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Abstract

This paper clarifies that in any society, when stricter penalties for tax evasion are sought based on the permeation of the idea that high-income earners should not be permitted to evade taxes for their personal gain, the government inadvertently creates a larger tax burden on the poor. In other words, a policy decision conferring preferential treatment or favoritism on the poor is incompatible with stricter penalties.

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1. Introduction

How should a government impose income tax? Since Mirrlees (1971), the standard answer to this question has been that the government, to secure the necessary income from tax revenue, should impose income taxes that maximize utilitarian economic welfare.

Under such a framework, Sato (2015) examined income tax structures by weighing the government's value judgment of favoritism toward the poor. That study examined cases in which fines for tax evasion were sufficiently low. However, in the present paper, I examine cases in which fines are sufficiently high. Sato (2015) found that sufficiently strong government favoritism toward the poor creates tax exemptions for the poor. In contrast, the results from the present study show that when sufficiently high fines are imposed on tax evaders, tax exemptions for the poor do not emerge despite the increase in bias toward the poor.

This tax exemption for the poor is the government's choice, or in other words, a complete self-realization by the government. The results of the present study show that governmental self-realization is affected by whether penalties are strict or lenient.

The remainder of this paper is organized as follows. In the next section, we set up a game theory model. In Section 3, we utilize the Nash equilibrium to derive the tax revenues envisioned by the government. In Section 4, we consider tax structures that would secure the necessary tax revenue. Finally, in Section 5, we summarize our conclusions.

2. Basic Model

The following three phases occur before tax revenue is collected by the government:

(i) All taxpayers file their income tax returns; at this point, a taxpayer's income level is known only to himself/herself.

(ii) Tax authorities conduct a random tax audit of taxpayers. During this audit, the taxpayer's earned income is revealed.

(iii) At this time, if the revealed income is different than the declared income, the tax authorities impose a fine on the taxpayer.

This shows that governmental tax revenue comprises income tax and fines. We will set up a model below to depict the three phases described above.

All taxpayers have either high income (I^H) or low income (I^L). The information on each taxpayer's income level is asymmetric between the taxpayer and tax authorities. In other words, all taxpayers know their own income level, but tax authorities only know the probability distribution: a particular taxpayer holds I^H (or I^L) with probability $q \in (0,1)$ (or $1-q$).

All taxpayers file their income tax returns under these circumstances. The income tax paid by individual taxpayers depends on the declared income level. In other words, when taxpayers file a return as I^H (or I^L), we expect the income tax paid to be T^H (or T^L), where, $T^L < T^H$.

At this juncture, if a taxpayer who owns I^H only declares I^L, it is possible for him/her to avoid income taxes with $T \equiv T^H - T^L$. In this study, in line with previous research on the theory of tax evasion (e.g., Greatz et al. 1986; Andreoni et al. 1998), we assume zero cost for the taxpayer to conceal his/her income by $I \equiv I^H - I^L$ to evade taxes.

Even if the taxpayer conceals part of his/her income, we assume that the concealed income will be found if the tax authorities spend C (investigation costs) on tax auditing. Then, the tax authorities may impose a fine F on the tax evader.

3. Nash Equilibrium

For simplicity, I assume that the taxpayer and tax authorities are both risk-neutral. I express the probability of tax evasion by the taxpayer to be α and the probability that the tax authorities will conduct tax auditing as β. The following equation shows the relation between the taxpayer's expected income (U^H) with respect to an arbitrary β:

$$U^H(\alpha,\beta)=\alpha\left(\beta\left(I^H-T^H-F\right)+(1-\beta)\left(I^H-T^L\right)\right)+(1-\alpha)\left(I^H-T^H\right). \qquad (1)$$

In contrast, by Bayes' rule, tax authorities can discover tax evaders with the probability

$$\mu=\frac{q\alpha}{q\alpha+1-q} \qquad (2)$$

,obtaining outcomes for $I \equiv I^H - I^L$. Therefore, the expected reward for the tax authorities (Π) is expressed in the following equation in relation to arbitrary α:

$$\Pi(\alpha,\beta)=\beta(\mu(\alpha)I-C). \qquad (3)$$

The partial derivative relating to α in Eq. (1) is linear with respect to β. As

$$\frac{\partial U^H(\alpha,1)}{\partial\alpha}=-F \qquad (4)$$

,by the intermediate value theorem, $\beta^0 \in (0,1)$; that is,

$$\beta^0=\frac{T}{T+F}. \qquad (5)$$

Now, the best response for a taxpayer with I^H is expressed by the following equation:

$$\alpha(\beta)\begin{cases} =1 & if \quad \beta < \beta^0 \\ \in [0,1] & if \quad \beta = \beta^0 \\ =0 & if \quad \beta > \beta^0 \end{cases} , \tag{6}$$

Similarly, we obtain the following best response of the tax authorities:

$$\beta(\alpha)\begin{cases} =1 & if \quad \mu(\alpha) > \mu^0 \\ \in [0,1] & if \; \mu(\alpha) = \mu^0 \\ =0 & if \quad \mu(\alpha) < \mu^0 \end{cases} , \tag{7}$$

where $\mu^0 = C / I$. Because probability μ has already been defined, we obtain the following:

$$\alpha^0 = \frac{(1-q)C}{q(I-C)} \quad . \tag{8}$$

When $I > C / q$, then $\alpha^o \in (0,1)$. Therefore, the best response of the tax authorities is represented by the following equation:

$$\beta(\alpha)\begin{cases} =1 & if \quad \alpha > \alpha^0 \\ \in [0,1] & if \; \alpha = \alpha^0 \\ =0 & if \quad \alpha < \alpha^0 \end{cases} \tag{9}$$

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The optimal reactions represented by Eqs. (6) and (9) show that if $I > C/q$, then the Nash equilibrium becomes the interior solution (i.e., a mixed strategy pair).

In the Nash equilibrium, the government's expected revenue (R^e) is represented by the following equation:

$$\begin{aligned} R^e &= q\alpha^0\beta^0\left(T^H + F\right) + (1-q)\beta^0 T^L \\ &\quad + \left(1-\beta^0\right)\left(q\alpha^0 + 1 - q\right)T^L + q\left(1-\alpha^0\right)T^H - q\alpha^0\beta^0 I. \end{aligned} \tag{10}$$

The sum from the first to fourth terms on the right-hand side is the expected tax revenue, consisting of income tax and fines. The fifth term is the expected reward granted by the government for the tax authority's outcome. In other words, Eq. (10) is the expected net revenue of the government.

Substituting $T^H = T + T^L$ and Eq. (5) into Eq. (10), we obtain the following:

$$R^e = \frac{T}{T+F} q\alpha^0\left(T^H + F - I\right) + \left(q\alpha^0 + 1 - q\right)T^L + q\left(1-\alpha^0\right)\left(T + T^L\right). \tag{11}$$

Rewriting the left-hand side as $R^e = \overline{R}$ (where $\overline{R}$ is a positive constant), we obtain the following:

$$\overline{R} = \frac{T}{T+F}\left(T + F - I\right) q\alpha^0 + T^L + q\left(1-\alpha^0\right)T\ . \tag{12}$$

To solve T^L, we obtain the following:

$$T^L = \overline{R} - \frac{qT}{T+F}\left(T + \alpha^0 I + F\right) \quad . \tag{13}$$

This equation represents a tax structure that generates the necessary tax revenue.

If we keep all other conditions the same (i.e., *ceteris paribus*) and differentiate T^L with T, we obtain the following:

$$dT^L / \mathrm{d}T = -q\frac{F}{\left(T+F\right)^2}\left(T + \alpha^0 I + F\right) - \frac{qT}{T+F} \quad , \tag{14}$$

where it can be seen that T^L is a decreasing function of T. Furthermore, if we differentiate $dT^L / \mathrm{d}T$ by T, we obtain the following:

$$d^2T^L / dT^2 = \frac{2qT\left(T+F\right)^2 + 2q\left(F\left(\alpha^0 I + F\right) - T^2\right)\left(T+F\right)}{\left(T+F\right)^4} \quad . \tag{15}$$

The sign of this equation is dependent on the sign of the second term of the numerator. Writing the formula as

$$\begin{aligned} \overline{U} &= F\left(\alpha^0 I + F\right) - T^2 \\ &= F^2 + \alpha^0 IF - T^2, \end{aligned} \tag{16}$$

we find that $\overline{U} > 0$ if F is sufficiently large. In this case, the set of tax structures $\left(T, T^{L}\right)$ leading to the necessary expected tax revenue is expressed in a strict convex function with respect to T .

4. Tax Structure

Of the tax structures shown in Section 3 that are capable of obtaining tax revenues, which should the government choose? The standard answer to this question is that the government should impose an income tax to maximize utilitarian economic welfare. To solve this problem, in this study, we define economic welfare as the honest tax filer's expected income (W). This is represented in the following equation:

$$W = (1-\alpha) q \left(I^{H} - T^{H}\right) + s (1-q) \left(I^{L} - T^{L}\right), \tag{17}$$

where s > 1 represents the government's degree of favoritism toward low-income earners. A similar idea is cited in Cramer and Gahvari (1995). As Sato (2015, p. 1) pointed out in their model, the government places a positive weight on the welfare of the high-income earners. When the weight is less than one, this study follows Cramer and Gahvari's economic welfare definition. However, Cramer and Gahvari's result is established under the condition that the weight is fixed unlike the weighting in this study.

Writing $W = \overline{W}$ (where $\overline{W}$ is a positive constant), substituting $T^H = T^L + T$, and solving T^L, we obtain the linear function with respect to T as follows:

$$T^L = \left(\hat{I} - \overline{W} - (1-\alpha)qT\right) / \left((1-\alpha)q + s(1-q)\right), \tag{18}$$

where $\hat{I} = (1-\alpha)qI^H + s(1-q)I^L$. The graph of Eq. (18) is in the first quadrant of the coordinates (T, T^L); as $\overline{W}$ becomes larger (smaller), it is positioned further southwest (northeast).

In Figure 1 below, $T^H > T^L > 0$ is in equilibrium. If we consider the coefficient of T as an increasing function of s as s increases, then T^L decreases but never becomes 0.

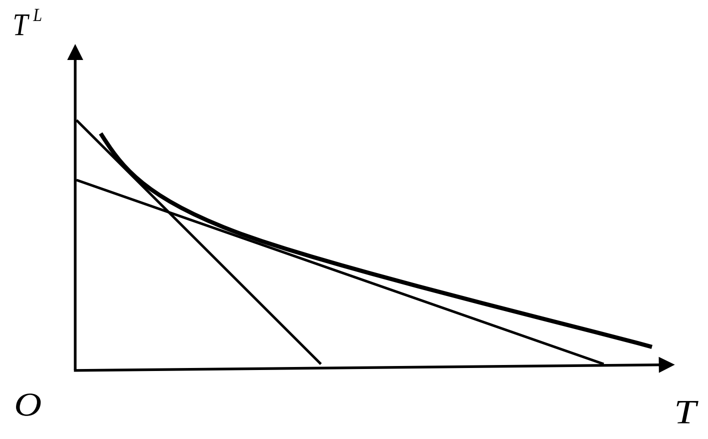

Figure 1. Optimal taxation

Therefore, the government can reduce the income tax imposed on low-income earners but cannot provide an income tax exemption to the poor.

5. Conclusions

This paper examines how a government determines income tax structures to derive the necessary tax revenue. The result obtained in this research infers the following point. In any society, when stricter penalties for tax evasion are sought based on the permeation of the idea that high-income earners should not be permitted to evade taxes for their personal gain, the government inadvertently creates a larger tax burden on the poor.

This result suggests that a policy decision conferring preferential treatment or favoritism on the poor is incompatible with stricter penalties.

Finally, we must be wary of the actions of governments who rely solely on income tax as their chief source of revenue. This is because, as pointed out by some authors, e.g., Zafer (2005) and Baunsgaard and Keen (2010), domestic taxes are not sufficient to be treated as the main financial resource of middle-income and developing countries; such countries must also rely heavily on trade tax revenues.

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