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Comparison of Environmental Policy Efficiencies in the Dominant Firm  
Model When Uncertainty Levels Differ Depending on Firms

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# **Comparison of Environmental Policy Efficiencies in the Dominant Firm Model When Uncertainty Levels Differ Depending on Firms**

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## **Abstract**

The present study examines the efficiencies of price regulation and quota in a dominant firm model based on the discussions of Weitzman (1974). Weitzman (1974) explained that the relative steepness of the slopes of marginal abatement cost (MAC) and marginal damage (MD) impacts the determinant condition of efficient policies under asymmetric information conditions. This study assumes a situation in which various levels of information asymmetry are observed by regulatory authorities between dominant firms and fringe firms. In a case where two types of firms—dominant and fringe—exist in the market and only dominant firms are the subject to environmental policies, a taxation policy will be efficient when the total MAC is equal to or greater than MD. Further, when the number of firms that are not subject to the regulation increases, the efficient environment policy depends on the level of uncertainty of each firm. When there is a relatively large (small) uncertainty for dominant firms, quota (price regulation) becomes a desirable policy.

## **1. Introduction**

Weitzman (1974) claimed that there is uncertainty between regulatory authorities and the firms subject to regulations and thus presented a theorem of efficient policy means. Weitzman's theorem showed that the relative steepness between the slope of marginal abatement cost (MAC) and that of marginal damage (MD) becomes an effective policy. In other words, if the slope of MAC is greater than that of MD, price regulation would become efficient, and if, by contrast, the slope of MAC is smaller than that of MD, quota (allocation of emission) becomes efficient.

Starting with this study by Weitzman (1974), several scholars have studied the efficiency of policy means by considering the existence of uncertainty, for example, Mandell (2008), Heuson (2010), Krysiak and Oberauner (2010), Ambec and Coria (2013), Mansur (2013), and Mori (2015, 2017). These studies show that the expansion of Weitzman's Theorem indicates the robustness of the initial model of Weitzman (1974).

The present study analyzed Mori (2015) and Mori (2017). Both these studies analyzed the

second-best environmental policy where environmental policy (e.g., environmental tax and quota) is applied only to dominant firms having control over price.<sup>1</sup>

The present study differs from Mori (2015) in two aspects: uncertainty and damage functions. In studies on the efficiency of environmental policy under uncertainty, it is assumed that regulatory authorities are uncertain with regard to firms' MAC and MD subject to the regulation. The present study, on the contrary, assumes that while there are two types of firms in the market, i.e., dominant firms and fringe firms, which behave as price takers, they both face different uncertainties. In other words, it is assumed that the estimation accuracies by regulatory authorities of the proprietary information with regard to the MAC of dominant and fringe firms largely differ from each other.

The second difference lies in the treatment of damage functions. Mori (2015) and Mori (2017) treated the contaminating materials generated by dominant and fringe firms as heterogeneous in nature.<sup>2</sup> The present study assumes that both dominant and fringe firms generate homogeneous contaminating materials in their production goods and establishes damage functions based on the total output.

Weitzman (1974), Mori (2015), and Mori (2017) have clarified that such uncertainty about damage functions does not impact the policy efficiency. Thus, the present study does not consider the uncertainty of damage functions. Instead, it focuses on the effect of the difference between the MAC of dominant firms and that of fringe firms on the policy efficiency.

The study results expand Weitzman's theorem. The traditional Weitzman's theorem states that when the slope of MAC is relatively flat, quota policy becomes a desirable policy. Yet, in the present study, when the number of fringe firms increases, MAC becomes smaller, but the choice of policy depends on the firms' uncertainty. In other words, when uncertainty of firms not subject to the policy is large, taxation policy is desirable. Therefore, it is clear that an efficient policy can be drawn by considering not only the information of dominant firms in the market but also the relative balance of information volume obtained from the innumerable existing small firms.

## 2. Model

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<sup>1</sup> In actual policies too, environmental tax and emission trading are applied only to energy-intensive firms and facilities. For the details of the grounds of the model, see Mori (2015) and Mori (2017).

<sup>2</sup> As for the assumptions of these damage functions, Duval and Hamilton (2002) and Roelfsema (2007) may be referred to.

A market is composed of one dominant firm and  $N$  number of fringe firms. The dominant firm is a price maker that determines the market price of goods, and fringe firms behave as price takers for whom the market price is a given. All of these firms produce homogeneous goods that emit contaminating materials in their production process, leading to environmental damages. Here,  $p^M = 1 - Q$  denotes the inverse market demand function.  $Q$  denotes the output of the market in the aggregate. The total supply function of the fringe firms is shown as  $q_F = bN(p - c - \theta_F)$ , where  $c(> 0)$  is a constant.  $q_F$  and  $p$  denote the outputs of fringe firms and price of goods, respectively, and  $\theta_F$  is the continuous stochastic variable, a parameter showing uncertainty with respect to the costs of fringe firms. In addition, the expectation value of  $\theta_D$  is zero, i.e.,  $E[\theta_F] = 0$ . From the supply function, we obtain  $MC_{tot}^F = c + \frac{1}{bN}q_F + \theta_F$  as the total marginal cost function of fringe firms. On the contrary, the marginal cost of dominant firms can be shown as  $MC_D = c + \theta_D$ , where  $\theta_D$  denotes the continuous stochastic variable, a parameter showing uncertainty with respect to the marginal costs of dominant firms. Further, the expectation value of  $\theta_D$  is zero, i.e.,  $E[\theta_D] = 0$ . Here, it is assumed that  $\theta_F$  and  $\theta_D$  are the variables independent of each other. Now,  $Q$ , the output of the market as a whole, can be expressed as  $Q = q_D + q_F$ . Therefore, when  $q_D = Q - q_F$  is solved for  $p$ , the inverse residual demand function  $p_D = \frac{1+bN(c+\theta_F)}{bN+1} - \frac{1}{bN+1}q_D$  is obtained.

$MAC_D$ , the marginal abatement cost of the dominant firm, can be obtained by subtracting  $MC_D$  from  $MR_D$  ( $= \frac{dp_D \cdot q_D}{dq_D}$ ):

$$MAC_D = \frac{1-c}{bN+1} - \frac{2}{bN+1}q_D + \frac{bN\theta_F - (bN+1)\theta_D}{bN+1} \quad (1)$$

$MAC_F$ , the marginal abatement cost of fringe firms, can be obtained by subtracting  $MC_{tot}^F$  from  $p_D$ , the market price determined by the dominant firm:

$$MAC_F = \frac{1-c-\theta_F}{bN+1} - \frac{1}{bN+1}q_D - \frac{1}{bN}q_F \quad (2)$$

By aggregating  $MAC_D$  and  $MAC_F$  with respect to their respective outputs, and by arranging it for  $p$ , the total marginal abatement cost  $MAC_{tot}$  is obtained:

$$MAC_{tot} = \frac{1-c}{(bN+1)} - \frac{2}{(2bN+1)}Q - \frac{bN}{(bN+1)(2bN+1)}\theta_F - \frac{1}{(2bN+1)}\theta_D \quad (3)$$

When the environmental damages caused by contaminating materials are expressed as damage function  $D(Q) = \frac{\lambda}{2}Q^2$ , then the marginal damage function,  $MD$ , is expressed as  $MD = \lambda Q$ . The socially optimal output level  $Q^*$  is one where  $MAC_{tot}$  and  $MD$  become equal:

$$Q^* = \frac{(1-c)(2bN+1)}{(bN+1)(2+\lambda(2bN+1))} - \frac{bN\theta_F + (bN+1)\theta_D}{(bN+1)(2+\lambda(2bN+1))} \quad (4)$$

The regulatory authorities cannot accurately capture  $Q^*$  because of the uncertainty of  $MAC$ . Therefore, they apply either taxation policy or quota on the dominant firm by making  $Q^E = E[Q^*]$ , a level where  $\theta_F = \theta_D = 0$  is achieved by excluding uncertainty as a second-best output level.

$$Q^E = \frac{(1-c)(2bN+1)}{(bN+1)(2+\lambda(2bN+1))} \quad (5)$$

### 3. Efficiency Loss Incurred

Here, assume that the regulatory authorities implement taxation policy on the dominant firm. The optimal tax rate  $t^*$  is determined by  $t^* = E[MAC_{tot}(Q^E)]$ .

The total output when taxation policy is implemented,  $Q^T$ , is obtained by aggregating  $q_D^t$  and  $q_F^t$ .  $q_D^t$  and  $q_F^t$  denote the outputs of dominant firm and fringe firms under tax regulation (see “Appendix”).

$$Q^T = q_D^t + q_F^t = \frac{(1-c)(2bN+1)(bN\lambda+1)}{(bN+1)(2+\lambda(2bN+1))} - \frac{bN}{2(bN+1)}\theta_F - \frac{1}{2}\theta_D \quad (6)$$

The total output when quota is implemented,  $\bar{Q}$ , is obtained by aggregating  $\bar{q}_D$  and  $\bar{q}_F$ .  $\bar{q}_D$  and  $\bar{q}_F$  denote the outputs of dominant firm and fringe firms under quota (see “Appendix”).

$$\bar{Q} = \bar{q}_D + \bar{q}_F = \frac{(1-c)(2bN+1)(bN\lambda+1)}{(bN+1)(2+\lambda(2bN+1))} - \frac{bN}{bN+1}\theta_F \quad (7)$$

The efficiency loss incurred by the implementation of the policy is determined using Equations (6) and (7). The efficiency loss due to taxation policy can be obtained by integrating the differences between the total  $MAC$  and  $MD$  from  $Q^T$  to  $Q^*$ . The expectation value of the efficiency loss is depicted in Equation (8):

$$E[EL_T] = \frac{\lambda^2(2bN + 1)(4b^2N^2(1 - c)^2 + b^2N^2\sigma_F^2 + (bN + 1)^2\sigma_D^2)}{8(bN + 1)^2(2 + \lambda(2bN + 1))} \quad (8)$$

$\sigma_F^2$  and  $\sigma_D^2$  denote the dispersion of uncertainties of the fringe firms and dominant firm, respectively. Similarly, the expectation value of the efficiency loss due to quota is shown as Equation (9).

$$E[EL_Q] = \frac{b^2N^2\lambda^2(1 - c)^2(2bN + 1)^2 + b^2N^2(1 + \lambda(2bN + 1))^2\sigma_F^2 + (bN + 1)^2\sigma_D^2}{2(bN + 1)^2(2bN + 1)(2 + \lambda(2bN + 1))} \quad (9)$$

#### 4. Results of Analysis

Equation (10) compares the expectation values of efficiency losses incurred because of taxation policy, on one hand, and quota, on the other:

$$E[EL_Q] - E[EL_T] = \frac{b^2N^2(2(1 + \lambda) + \lambda(2bN + 1))\sigma_F^2}{8(bN + 1)^2(2bN + 1)} + \frac{(2 - \lambda(2bN + 1))\sigma_D^2}{8(2bN + 1)} \quad (10)$$

Quota will be efficient if the value of Equation (10) is smaller than zero, and the taxation policy will be efficient if it is greater than zero. When the value of Equation (10) equals zero, then the efficiencies of both the policies will be the same. Owing to the definitions of the model, the determinants of the preferred policy depend on the numerator of the second term in Equation (10). When the second term of the Equation (10) is zero, in other words, when  $2 - \lambda(2bN + 1) = 0$ , the absolute values of the slopes of MD and total MAC are the same.

It is known that, in Weitzman (1974), the efficiencies of taxation policy and quota are equal when the slopes of MD and total MAC are the same. In the present study though, when the slopes of MD and total MAC are the same at  $(2 - \lambda(2bN + 1) = 0)$ , Equation (10) takes a positive value and the taxation policy becomes the desirable one. This is a result that expands the conventional Weitzman's theorem.<sup>3</sup>

In addition, when  $2 - \lambda(2bN + 1) > 0$ , the slope of MD will be smaller than that of total MAC. In such a case,  $E[DWL_Q] > E[DWL_T]$  is established and taxation policy becomes preferable.

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<sup>3</sup> This result can also be confirmed in Mori (2015). While this paper is somewhat different from Mori (2015) in the treatment of uncertainty and setting of damage function, in dominant firm models where only dominant firms are subject to a policy, taxation policy can have universalness when the slopes of MD and total MAC are equal

This result is consistent with the result obtained by the traditional Weitzman's theorem.

Here, consider the relationship between the number of firms and the efficiency of policy. Equation (11) shows that the comparison of the expectation values of efficiency losses in the case of the number of fringe firms,  $N$ , increases indefinitely:

$$\lim_{N \rightarrow \infty} E[EL_Q] - E[EL_T] = \frac{\lambda(\sigma_F^2 - \sigma_D^2)}{8} \quad (11)$$

The following proposition is derived from Equation (11).

**Proposition:** In cases where only dominant firms are subject to a policy and the number of fringe firms increases, taxation policy becomes preferable in case of  $\sigma_F^2 > \sigma_D^2$ , and quota becomes effective in case of  $\sigma_F^2 < \sigma_D^2$ .

When uncertainties regarding the firms' MAC differ, if the uncertainty of the firms subject to a regulation is relatively large, the expectation value of efficiency loss will be smaller in case quota is implemented. In contrast, if the uncertainty of the firms that are not subject to the policy is relatively large, applying taxation policy will be desirable.

As confirmed in Equation (3), as the number of fringe firms increases, the slope of the total MAC becomes less steep. This, of course, implies that as the number of firms increases, the abatement costs are more equally divided among the fringe firms. Under Weitzman (1974) as well as under Mori (2015) and Mori (2017) that adopt the dominant firm model, quota will become the desired policy if the slope of total MAC is relatively gentle.

Equation (16) indicates that efficient policy is underspecified by quota alone but is determined by the relative size of uncertainty. Although the dominant firm may have a nearly 100% market share, the output share of fringe firms and that of the dominant firm are two sides of the same coin, so the dominant firm must be vigilant about the behaviors of fringe firms.

When the uncertainty of the dominant firm is relatively large, or ( $\sigma_F^2 < \sigma_D^2$ ), then quota becomes the desirable policy. Quota, in comparison with price regulation, allows regulatory authorities to achieve with certainty the level they judge desirable. Therefore, when the uncertainty of the dominant firm is large, implementing quota will not excessively hinder the business activities of fringe firms, and the reduction of environmental damages would be more secured.

If, on the contrary, the uncertainty of the dominant firm is relatively small ( $\sigma_F^2 > \sigma_D^2$ ), taxation policy is more desirable. If the regulatory authorities can gather information on the abatement cost of the dominant firm more accurately, then implementing price regulation would be a better option as it does not impede efficiency. As fringe firms decide their own output levels based on the price determined by the dominant firm, taxation policy will be more efficient as it will not distort the efficiency brought about by competition. Therefore, in the implementation of an environmental policy, the present study explains that considering the estimation accuracy on abatement cost, not only of the firms subject to that policy but also those that are not subject to it, becomes a condition for decision making on the efficient policy.

## 5. Conclusion

The present study compares the efficiencies of policies, when regulatory authorities have uncertainties about the firms, by primarily expanding the Mori (2015) model. It differs from prior studies in the following two points. The first is the treatment of uncertainty. The present study assumed that gaps are different between the MAC of firms subject to the policy of the regulatory authorities and that of the firms that are not subject to such policy. The other difference is in the treatment of damage function. Mori (2015) and Mori (2017) treat the contaminating materials generated by dominant firms and those by fringe firms as heterogeneous in nature. The present study, instead, assumes that all the firms generate homogeneous contaminating materials and determine damage function based on its total volume.

The dominant firm model was expanded into a more general one under the present study as compared with previous studies, and through its analysis, the following two results were obtained. If total MAC and MD are equal, taxation policy would become more efficient and when the number of fringe firms not subject to the policy increases, policy efficiency depends on the relative degree of uncertainty.

The analysis also revealed that regulatory authorities are unable to implement efficient policy solely based on information about the firms that are subject to the policy. When uncertainties differ between the firms contingent to the implementation of the policy, the determination of the policy depends on how accurately the regulatory authorities can capture the proprietary information of the respective groups. The present study treated uncertainty as a parameter of cost function, and it would be possible to define such a parameter in more detail, which should remain a research subject for future studies.



## Appendix

The optimal tax rate  $t^*$  is determined by  $t^* = E[MAC_{tot}(\bar{Q})] = \frac{\lambda(1-c)(2bN+1)}{(bN+1)(2+\lambda(2bN+1))}$ . When taxation policy is implemented, the dominant firm will produce at the level  $q_D^t$ , which would make  $MAC_D = t^*$ .

$$q_D^t = \frac{1-c}{2+\lambda(2bN+1)} + \frac{bN}{2}\theta_F - \frac{bN+1}{2}\theta_D \quad (\text{A.1})$$

In case of quota,  $\bar{q}_D$ , the output that the dominant firm decides is the one that would make  $E[MAC_D] = t^*$ .

$$\bar{q}_D = \frac{1-c}{2+\lambda(2bN+1)} \quad (\text{A.2})$$

The output that fringe firms decide is determined. In case the taxation policy is implemented for the dominant firm, the fringe firms will determine the output  $q_F^t$ , which would make  $MAC_F = 0$ . Note here that  $q_D = q_D^t$ :

$$q_F^t = \frac{bN(1-c)(1+\lambda(2bN+1))}{(bN+1)(2+\lambda(2bN+1))} - \frac{bN(bN+2)}{2(bN+1)}\theta_F + \frac{bN}{2}\theta_D \quad (\text{A.3})$$

Similarly, the output of fringe firms is determined when quota is adopted. In this case,  $q_D = \bar{q}_D$ :

$$\bar{q}_F = \frac{bN(1-c)(1+\lambda(2bN+1))}{(bN+1)(2+\lambda(2bN+1))} - \frac{bN}{bN+1}\theta_F \quad (\text{A.4})$$

The total output when taxation policy is implemented,  $Q^T$ , is obtained by aggregating Equations (A.1) and (A.3). The total output when quota is implemented,  $\bar{Q}$ , is obtained by aggregating Equations (A.2) and (A.4).

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